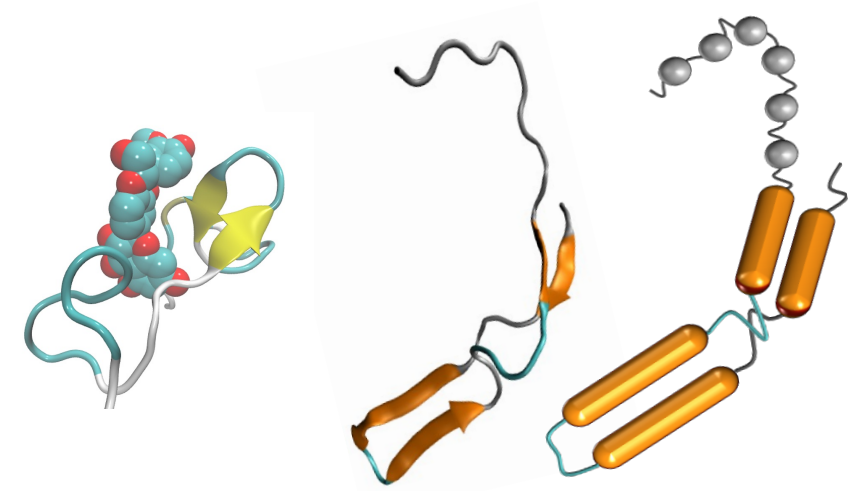
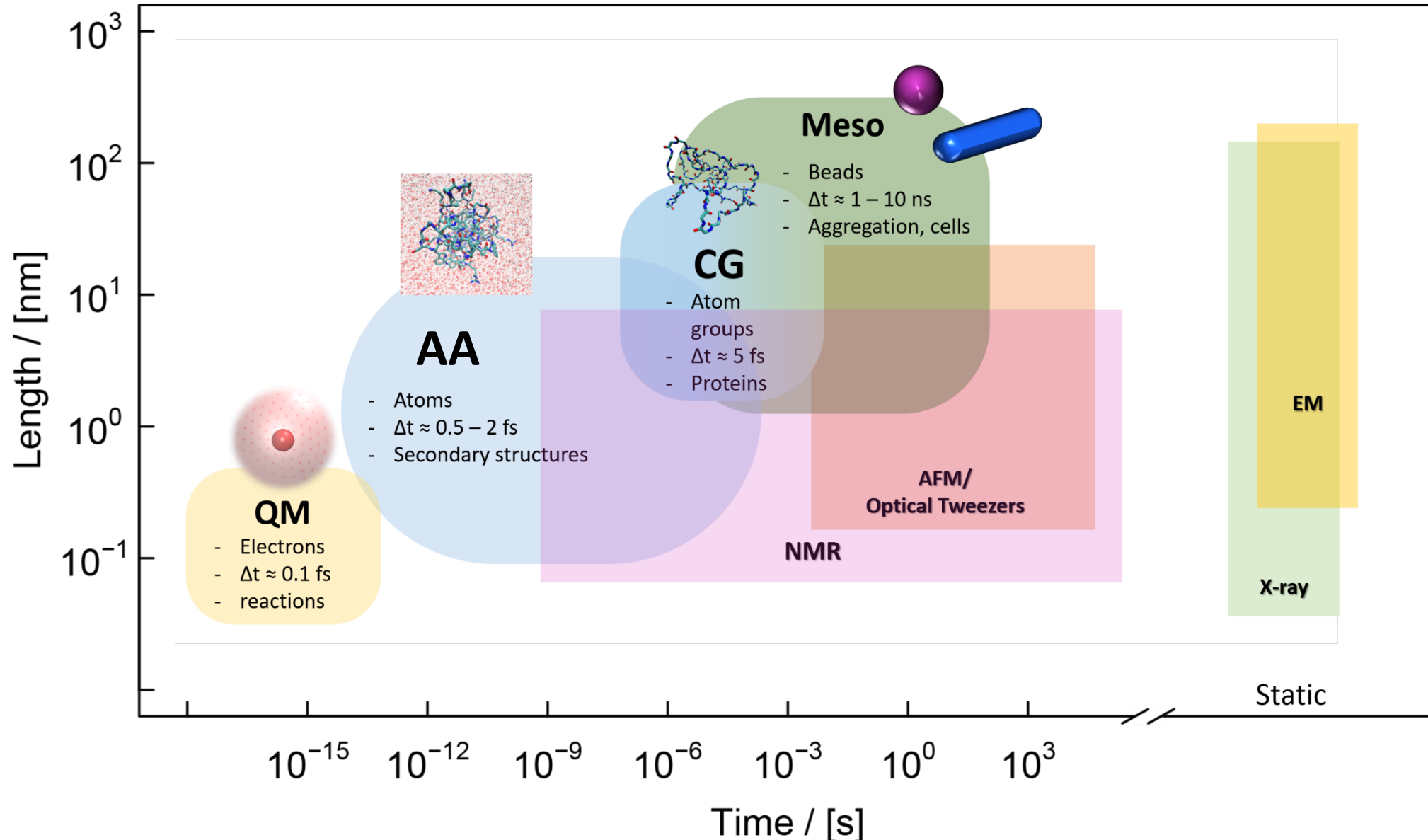


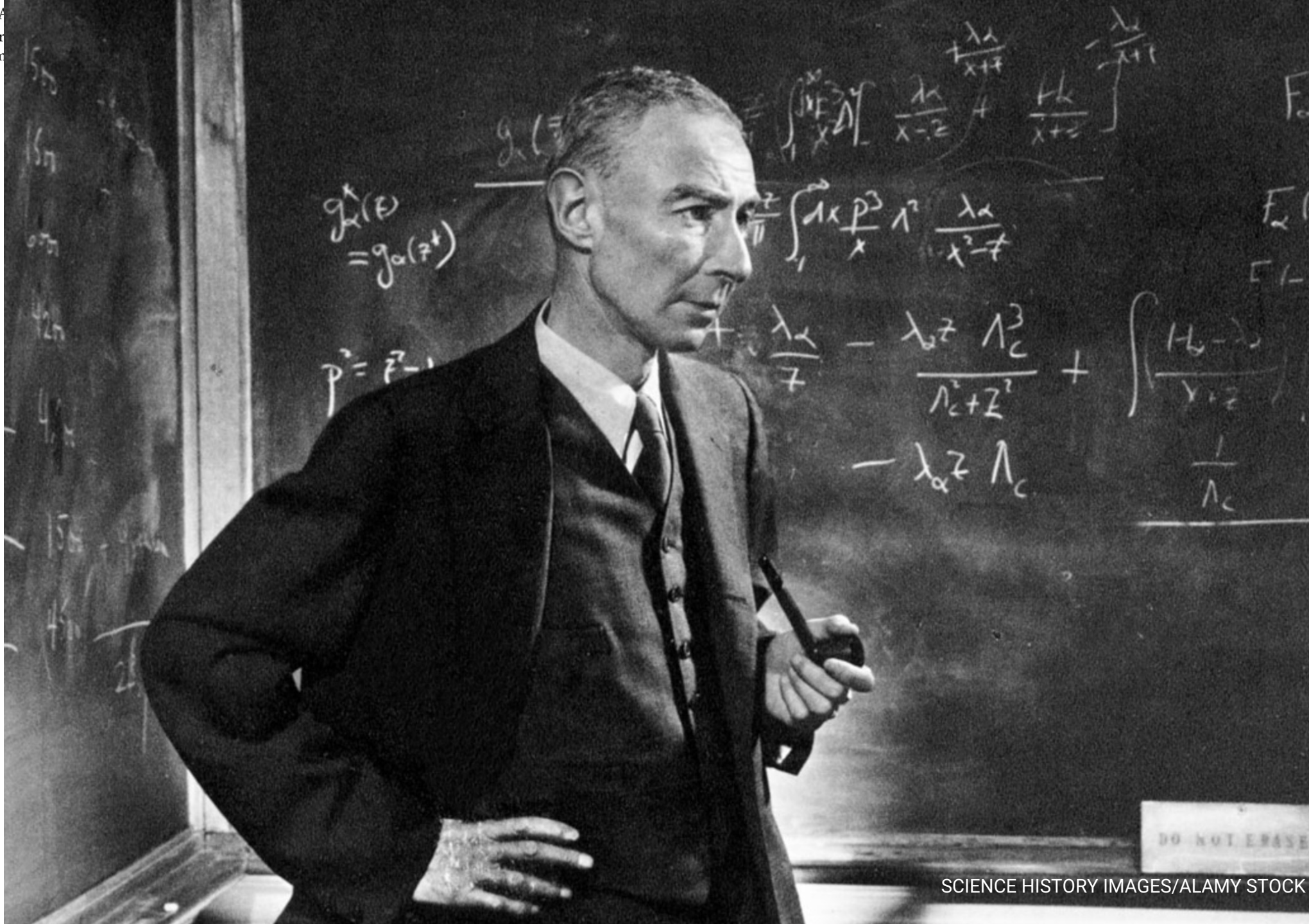
Coarse-graining

Why? What? How?



Why? What?





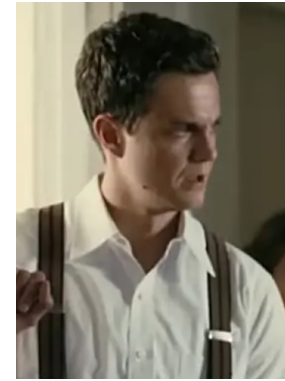


Robert J. Oppenheimer

1904 – 1967

“Father of the atomic bomb”

Born-Oppenheimer approximation (1927)
electronic motion and nuclear motion in molecules can be separated

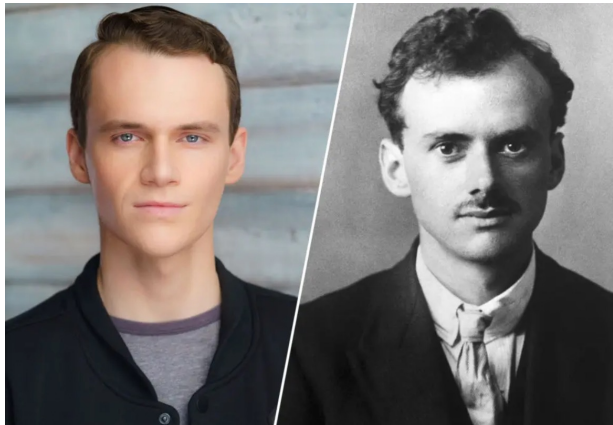


Richard Feynman

1918 – 1988

1965 – Nobel price
development of quantum electrodynamics

1959 - designing
miniaturized machines



Paul Dirac

1902 – 1984

1933 – Nobel price
discovery of new productive forms of atomic theory



Erwin Schrödinger

1887 – 1961

1933 – Nobel price
discovery of new productive forms of atomic theory

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle$$



Niels Bohr

1885 – 1962

1922 – Nobel price
investigation of the structure of atoms and the radiation emanating from them

Developed the model for the atom



Werner Heisenberg

1901 – 1976

1932 – Nobel price
creation of quantum mechanics

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$



Before CG



2020

AlphaFold & ML

2013



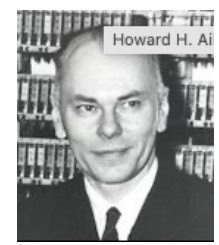
Walter Kohn



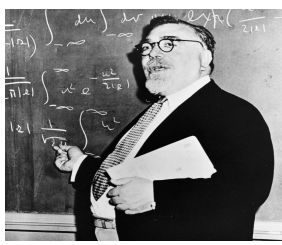
John Pople

Density functional theory & Computational methods in quantum chemistry

1966



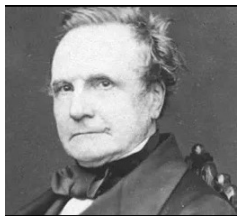
Howard H. Aiken



Norbert Wiener



Alan Turing



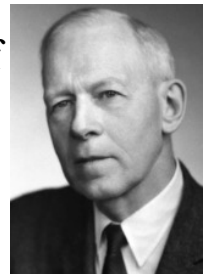
Charles Babbage
(1791-1871)
Analytical Machine

1930s

H, He, H₂

Multiscale models for complex chemical systems

1998



Robert Mulliken

1940-1950s
Analytical Engine
ENIAC



Martin Karplus



Michael Levitt



Arieh Warshel

Molecular orbital theory & its use in the calculation of electronic structure of molecules

the era of computing chemists, when hundreds if not thousands of chemists will go to the computing machine instead of the laboratory for increasingly many facets of chemical information, is already at hand.



Niels Bohr



Erwin Schrodinger



Paul Dirac



Werner Heisenberg

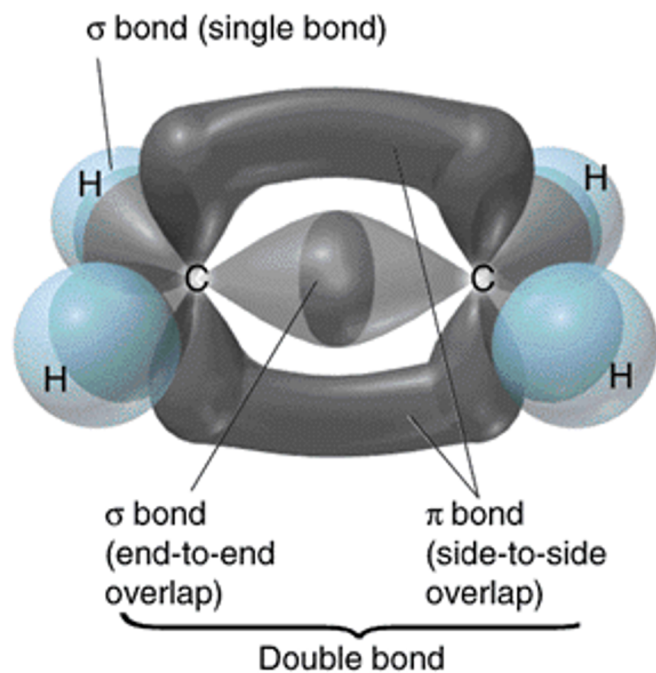
1959



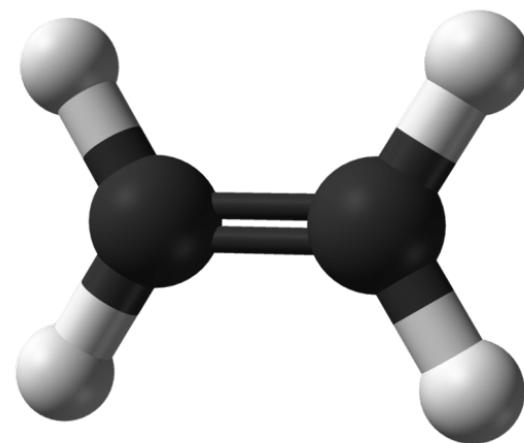
~ 1970



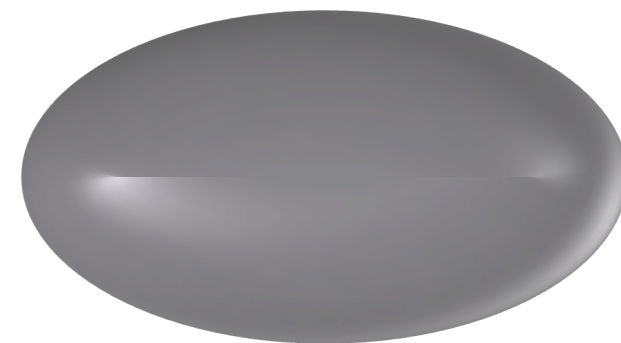
The basics – structural resolution



Ethene in electronic structure level representation (orbitals)

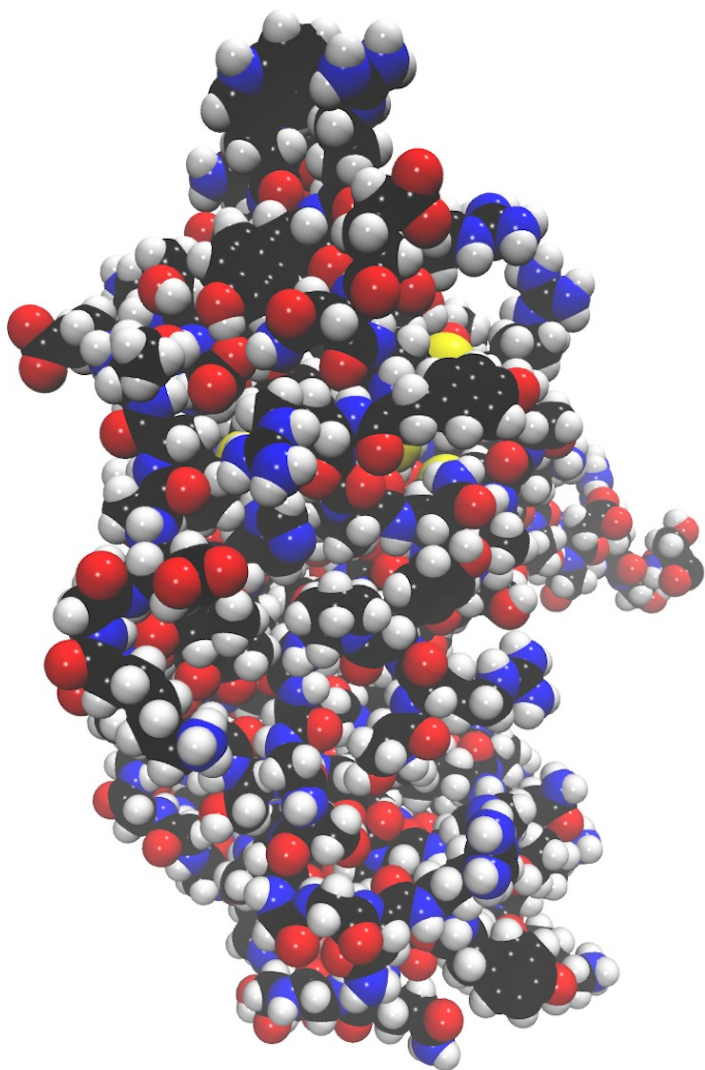


Ethene in all-atom molecular mechanics representation (spheres)



Ethene in coarse-grained representation

Molecular Mechanics Representation



- Atom \rightarrow sphere
- Biomolecular force field: shapes the collection of spheres into things that look like molecules through so-called **bonded** interactions
- Force field: atom-atom interactions of distal parts through **nonbonded** interactions.
- Electronic structure is in general coarse-grained out and expected to be captured by the force field.



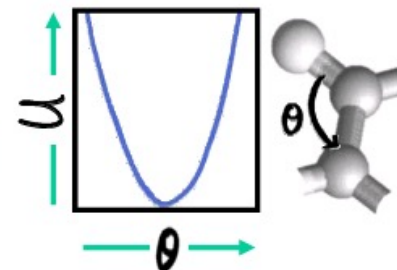
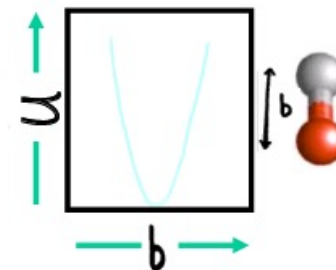
The basics on one slide (classical simulations)

$$V = \sum \frac{1}{2} K_b (b - b_0)^2 + \sum \frac{1}{2} K_\theta (\theta - \theta_0)^2$$

All Bonds

All Angles

Hooke 1635



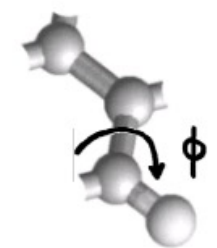
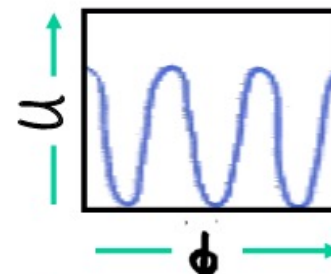
$$F = m \frac{d^2 r}{dt^2} = ma$$

$$F = - \frac{dV(r)}{dr}$$

$$+ \sum K_\phi [1 - \cos(n\phi + \delta)]$$

All Torsion Angles

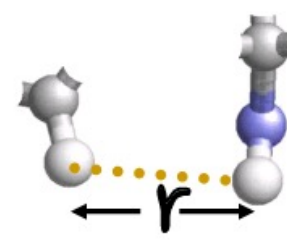
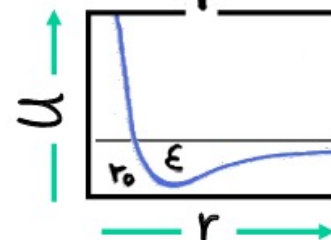
Fourier 1768



$$+ \sum \epsilon \left[\left(\frac{r_0}{r} \right)^{12} - 2 \left(\frac{r_0}{r} \right)^6 \right]$$

All Nonbonded pairs

Van der Waals 1837

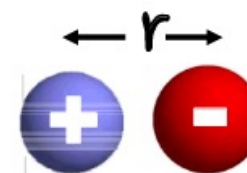
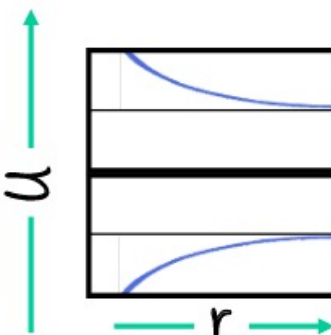


$$+ \sum \frac{332 q_i q_j}{r}$$

All partial charges

Coulomb 1736

Simple sum over many terms



How does it work?

- The force field is an analytical function of the spatial coordinates of the atomic nuclei
- Several different force fields are commonly available (CHARMM, AMBER, OPLS)

$$F = m \frac{d^2 r}{dt^2} = ma$$

Given the potential, one can numerically integrate the trajectory of the whole system as a function of time.

$$F = - \frac{dV(r)}{dr}$$

The force F is given by the gradient of the potential $V(r)$.

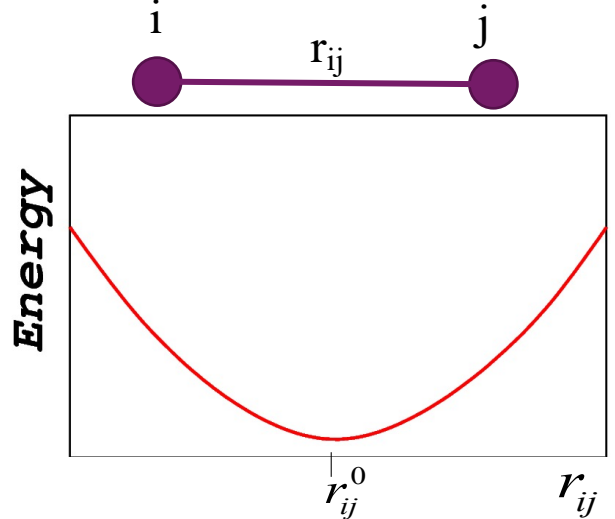
$$V = V_{\text{bonds}} + V_{\text{angles}} + V_{\text{dihedrals}} + V_{\text{nonbonded}}$$

$$V = V_{\text{bonds}} + V_{\text{angles}} + V_{\text{dihedrals}} + V_{\text{nonbonded}}$$

the energy needed to **stretch** a covalent bond between two atoms by Hooke's law for the potential energy stored in a spring

$$V_{\text{bonds}}(r_{ij}) = k_{ij}^B (r_{ij} - r_{ij}^0)^2$$

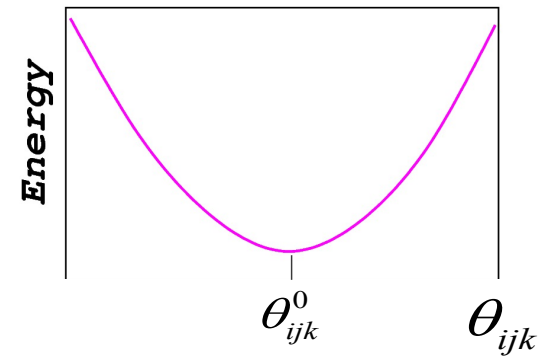
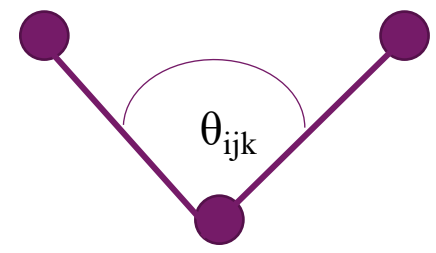
k_{ij}^B - constant (bond stiffness)
 r_{ij}^0 - equilibrium distance



the energy needed to **bend** the angle formed by **two** covalent bonds

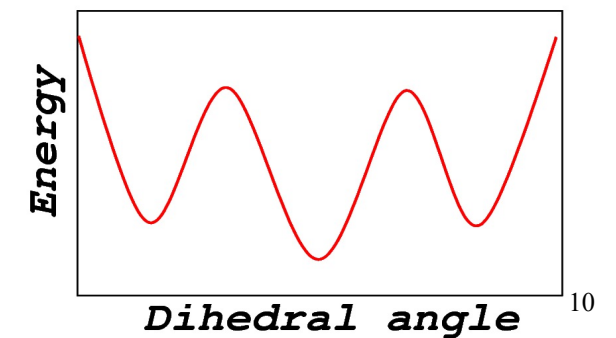
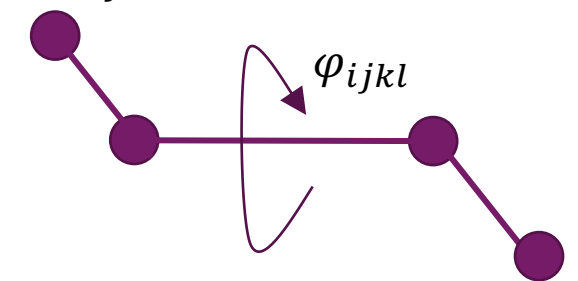
$$V_{\text{angles}}(\theta_{ijk}) = k_{ijk}^\theta (\theta - \theta_{ijk}^0)^2$$

k_{ij}^θ - constant (angle rigidity)
 θ_{ijk}^0 - equilibrium angle width

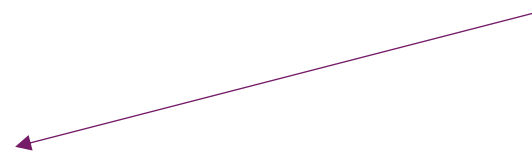


the energy needed to **bend** the dihedral angle formed by **three** covalent bonds

$$V_{\text{dihedrals}}(\varphi_{ijkl}) = k_{ijkl}^\varphi [1 + \cos(n\varphi_{ijkl} - \delta)]$$



$$V = V_{\text{bonds}} + V_{\text{angles}} + V_{\text{dihedrals}} + V_{\text{nonbonded}}$$



for non-covalently bound atoms

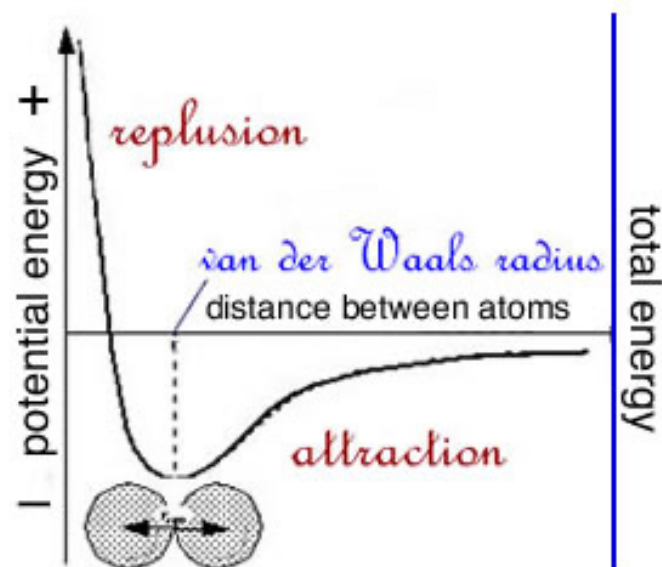
$$V_{\text{vdW}} + V_{\text{electrostatic}}$$

$$V_{\text{vdW}} = 4\epsilon \left[\left(\frac{\sigma}{r_{ij}} \right)^{12} - 2 \left(\frac{\sigma}{r_{ij}} \right)^6 \right]$$

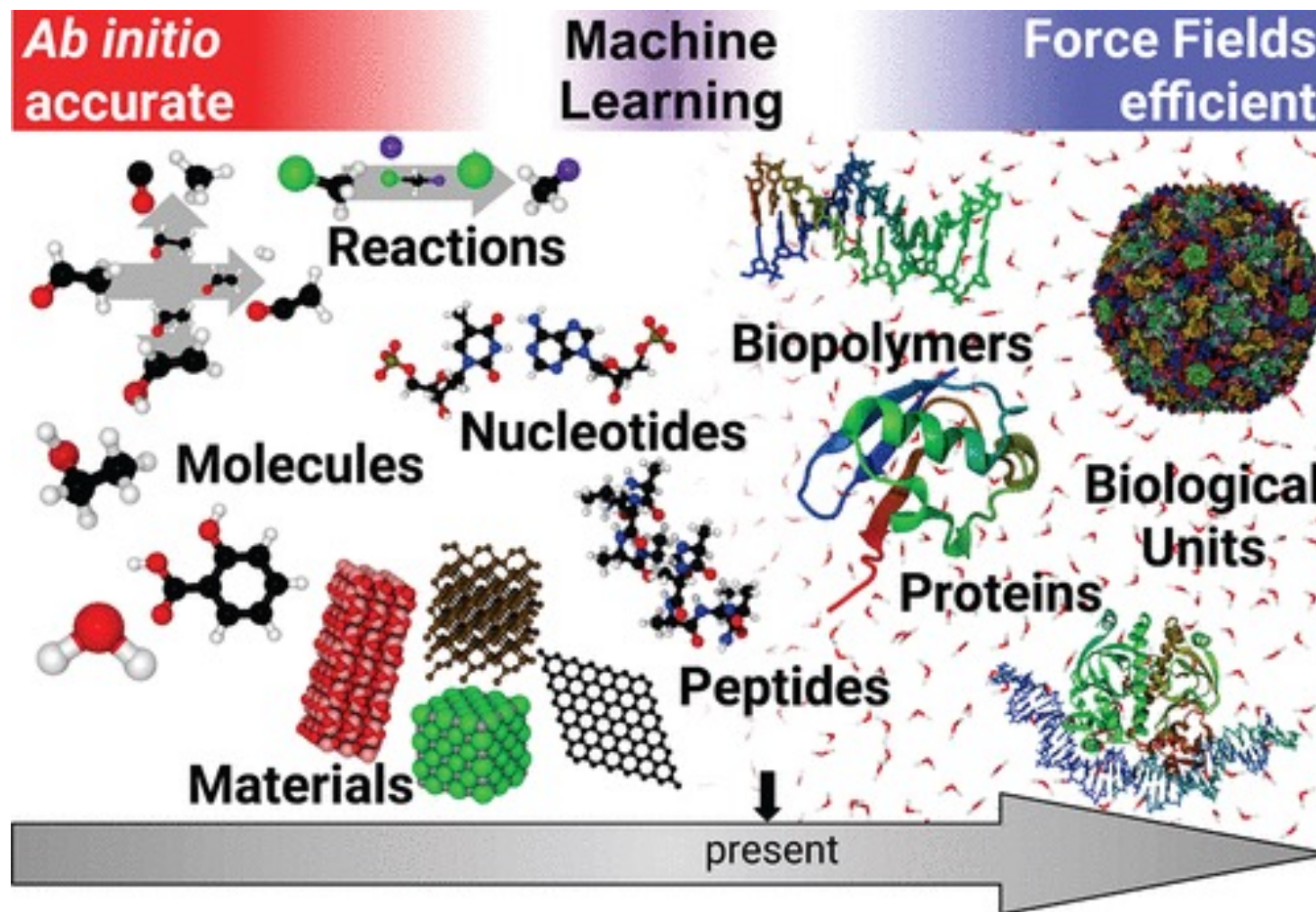
$$V_{\text{electrostatic}} = \frac{q_i q_j}{\epsilon_d r_{ij}}$$

ϵ_d - dielectric constant of the surrounding medium

q_i, q_j - charges



The dream!





Intermezzo – Why?



CONVEX SUPERCOMPUTER - DATA PROCESSOR (1989, U.S.A)

The Convex Company was founded in 1982, with a view to creating supercomputers for the technical and scientific field. Similar to the Cray computers, the Convex is based on vectors or a system that is parallel with the vector-type registers, thus reducing the bandwidth of the available memory for each processor to a single operation per cycle. Convex invested massive amounts of money into automatic vectorization techniques.

The Convex C1 is a vector-type computer with a single processor, which came out in 1985. The Convex supercomputer runs with the UNIX operating system, the OS version, called Convex OS, and it is equipped with type C automatic parallelizing compilers and Fortran.

Technical specifications:

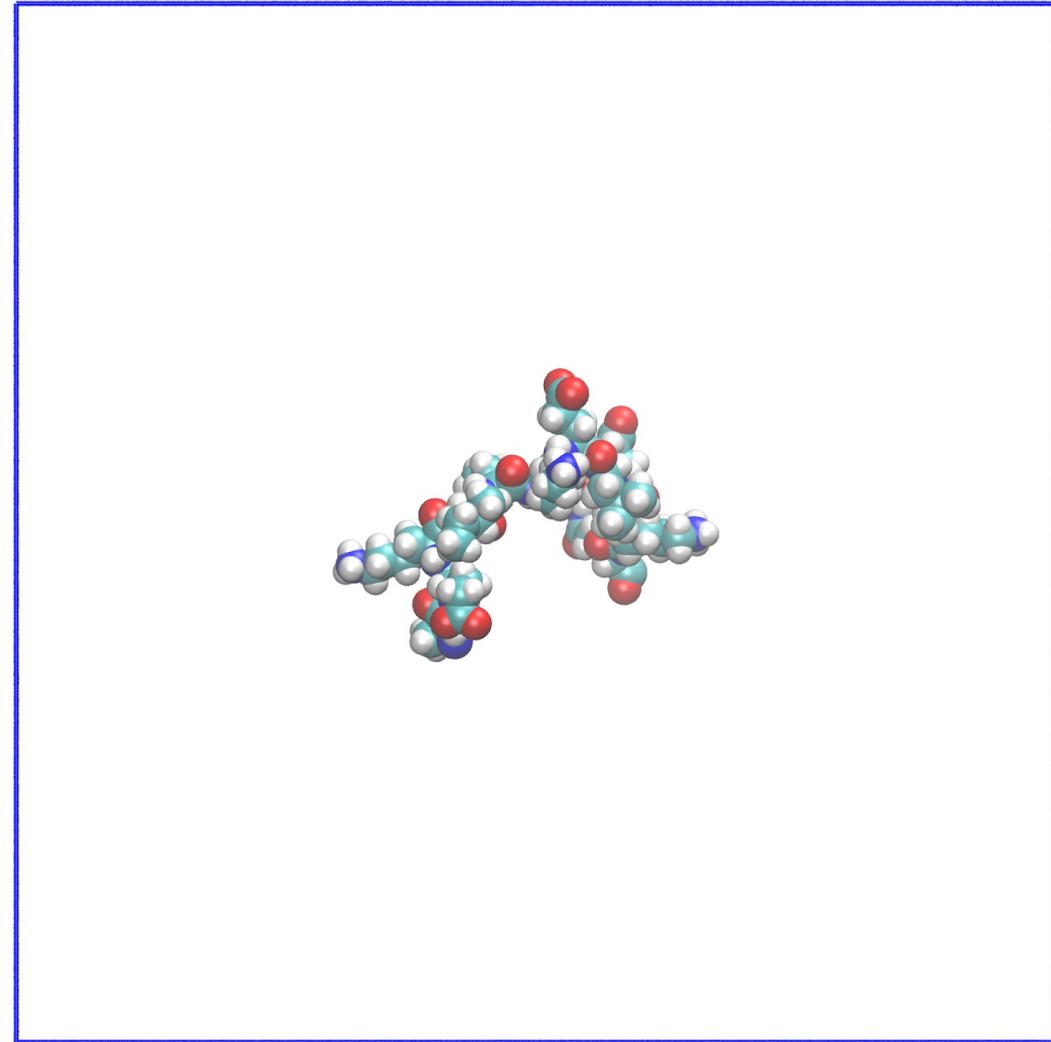
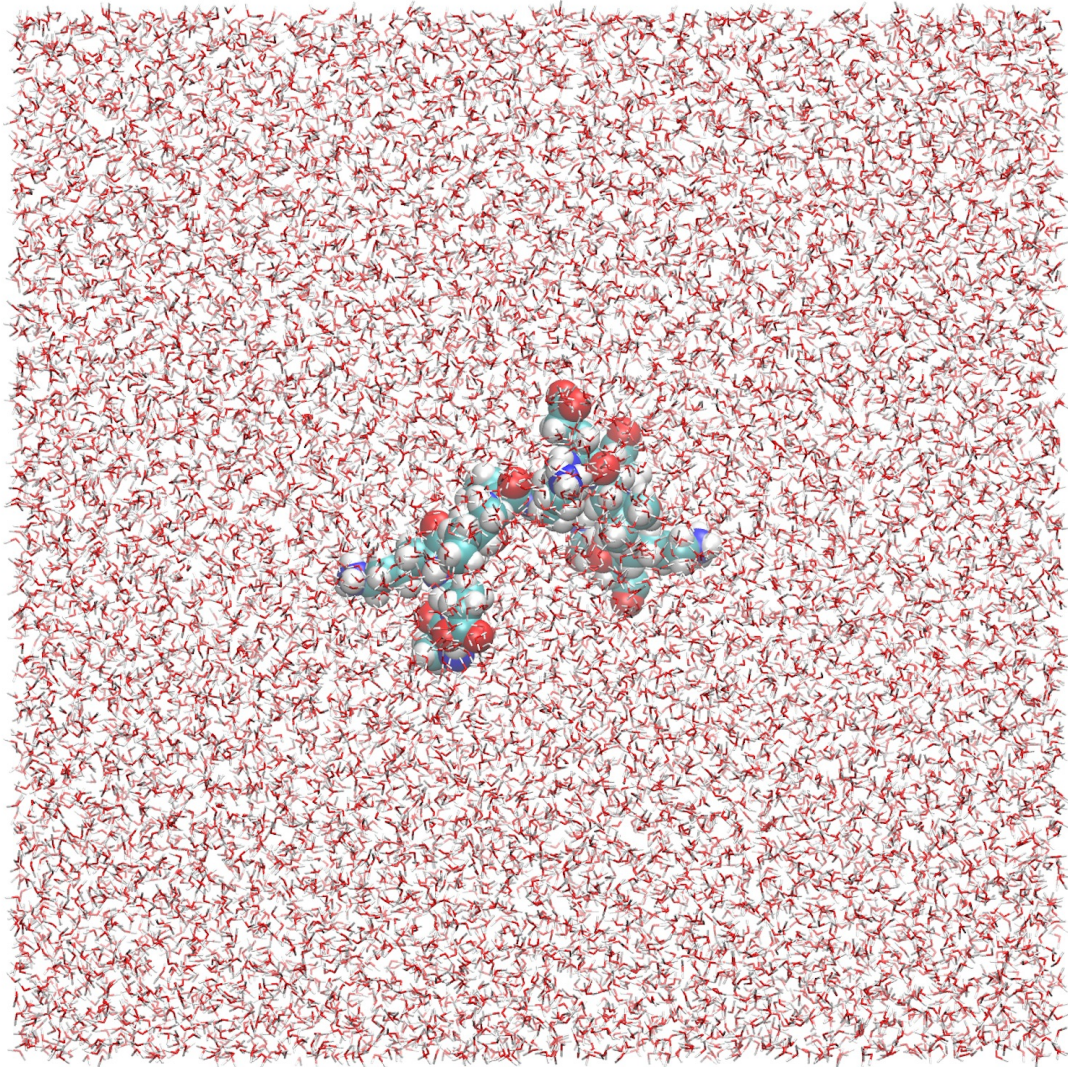
Hardware: C120 Processor built on 64 bytes with integrated scalar functions and vectorial functions.

Memory system: 4 GB virtual memory and 1 GB physical memory.

Full system	76,832 cores + 144 GPUs: 6.1 Pflop/s (peak performance)
Thin nodes (AMD Rome 7H12)	64,512 cores: 2.69 Pflop/s
Fat nodes (AMD Rome 7H12)	9,216 cores: 394 Tflop/s; also 6.4 TB of NVMe local storage
GPGPU nodes (Intel IceLake + 4x Nvidia A100)	2,592 cores + 144 GPUs: 3.0 Pflop/s
High-memory nodes (AMD Rome 7H12)	512 cores: 22 Tflop/s; 4/8 TB per node
Memory	245 TB memory (CPU + GPU/HBM)
Disk space	720 TB home file systems, 12.4 PB scratch and project; 200TB NVMe parallel filesystem

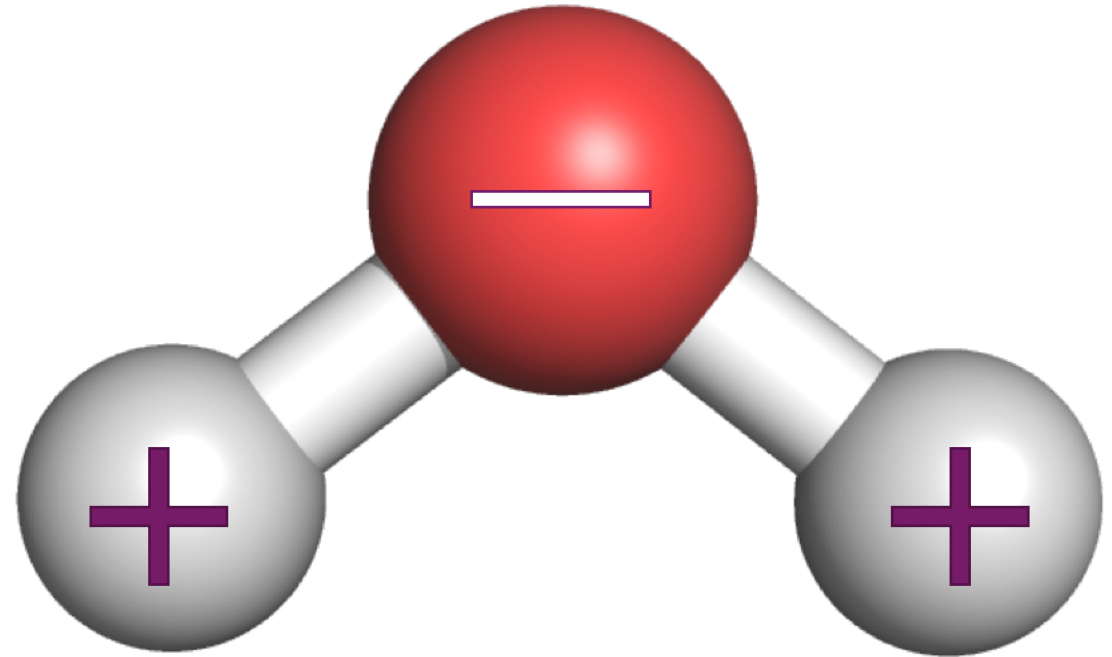


How does it work?

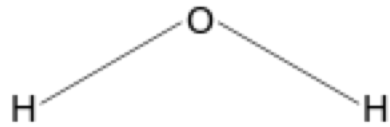


Water models

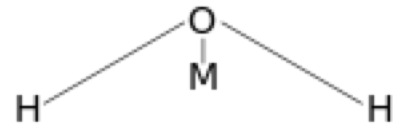
- **Rigid**
 - Fixed atom positions
 - Only non-bonded interactions
- **Flexible**
 - Atoms on “springs”
 - Include bond stretching and angle bending
 - Reproduce vibration spectra
- **Polarizable**
 - Include specific polarization terms



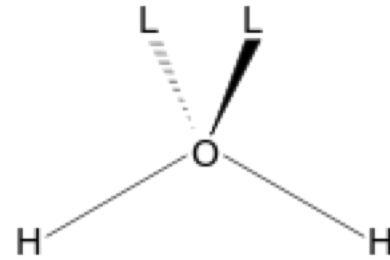
Rigid water models



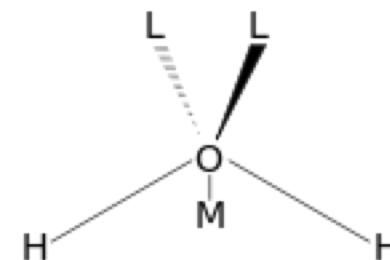
3-site



4-site



5-site

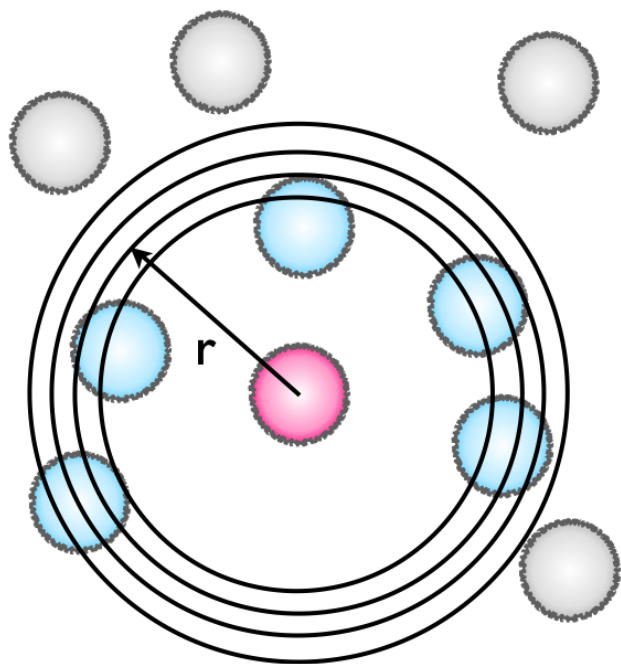


6-site

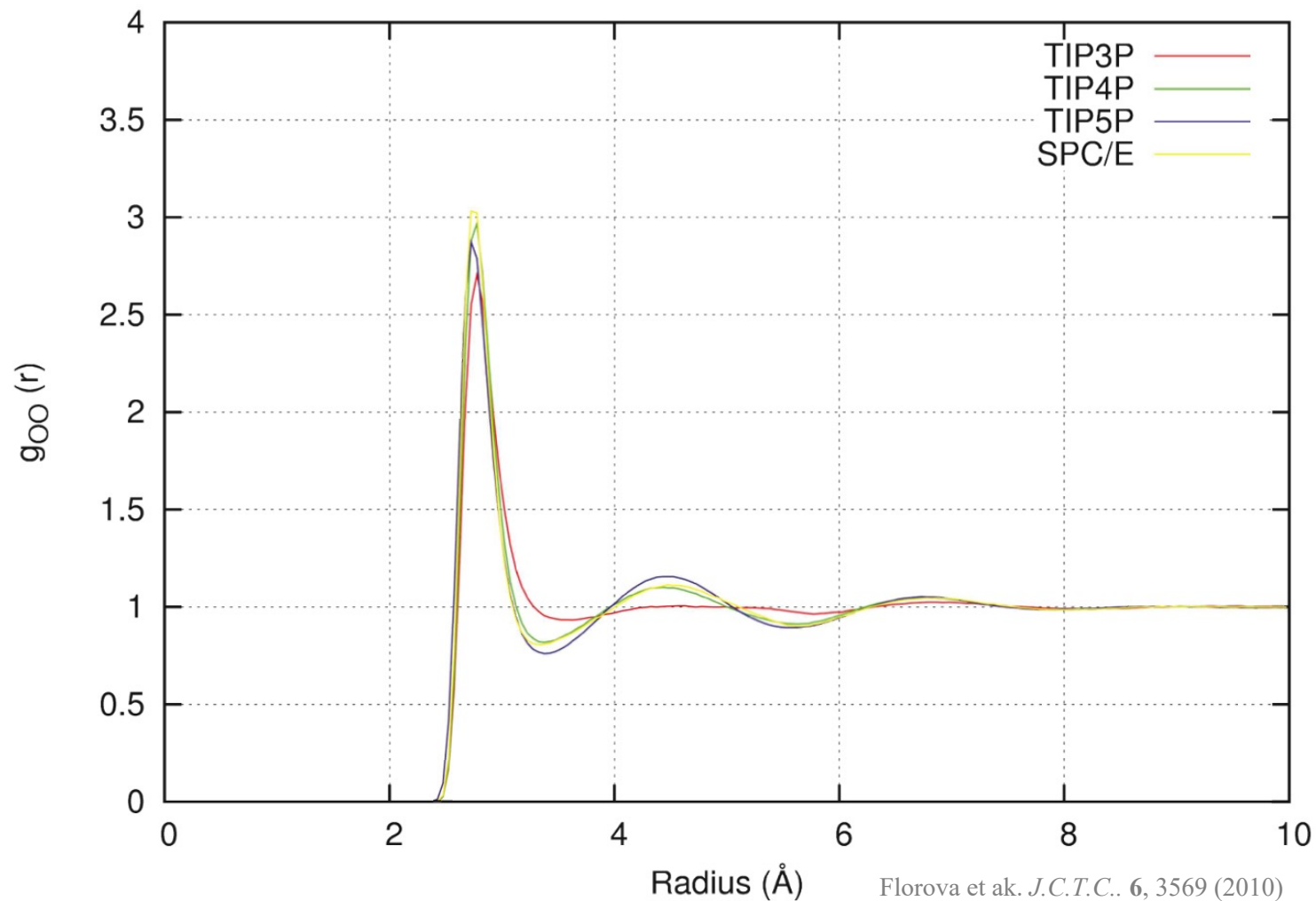
$$V = \sum_{pairs} \left(\underbrace{\frac{A_{LJ}}{r_{O-O}^{12}} - \frac{B_{LJ}}{r_{O-O}^6}}_{\text{LJ for O-O}} + \underbrace{k \frac{q_i q_j}{r_{ij}}}_{\text{Coulomb}} \right)$$

r_{O-O} oxygen-oxygen distances
 A_{LJ}, B_{LJ} Lennard-Jones parameters
 r_{ij} distance between charged sites
 k constant in Coulomb's law

Radial distribution function $g(r)$

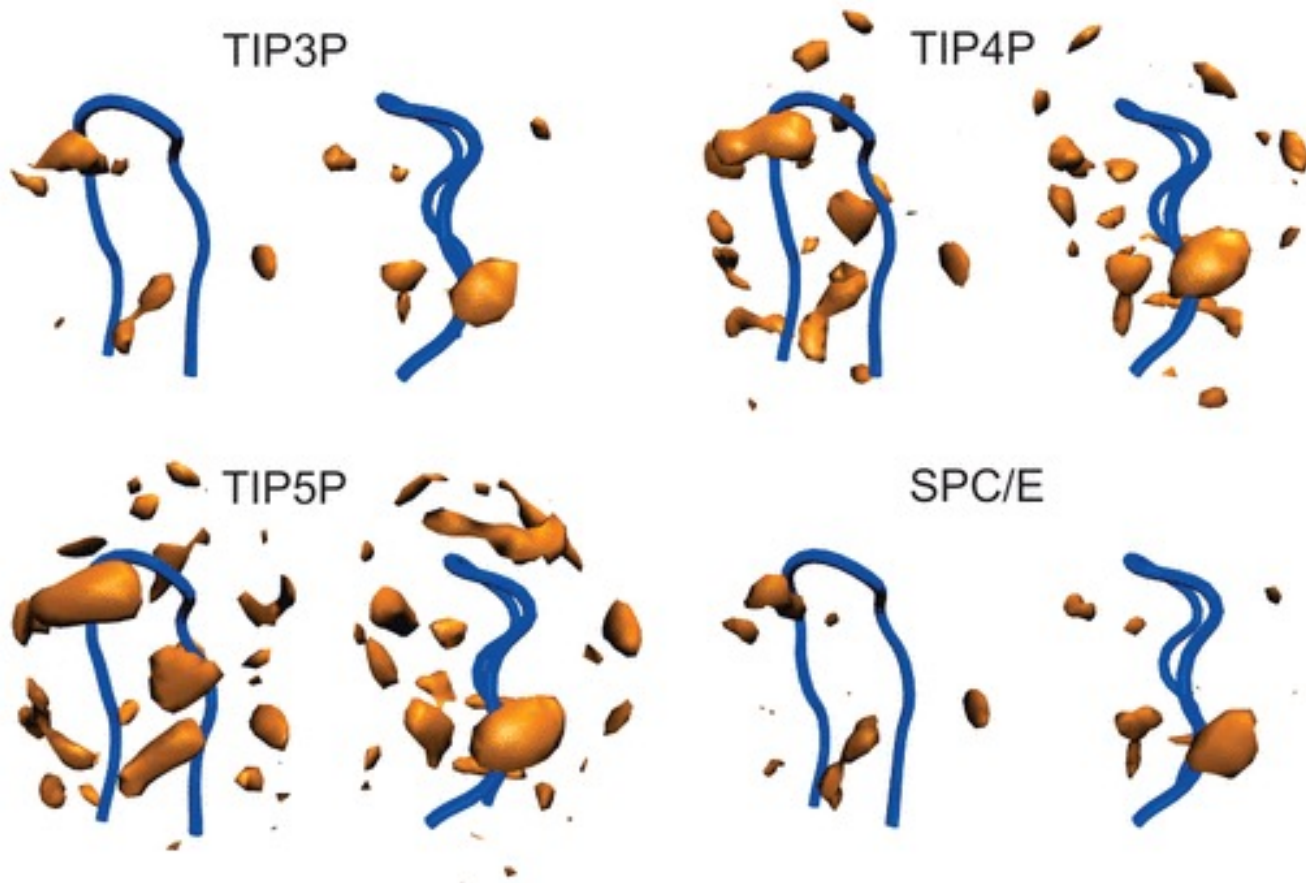


Small differences in geometry, charges
 but huge differences in water structure and
 dynamics



Florova et al. *J.C.T.C.* **6**, 3569 (2010)

Example

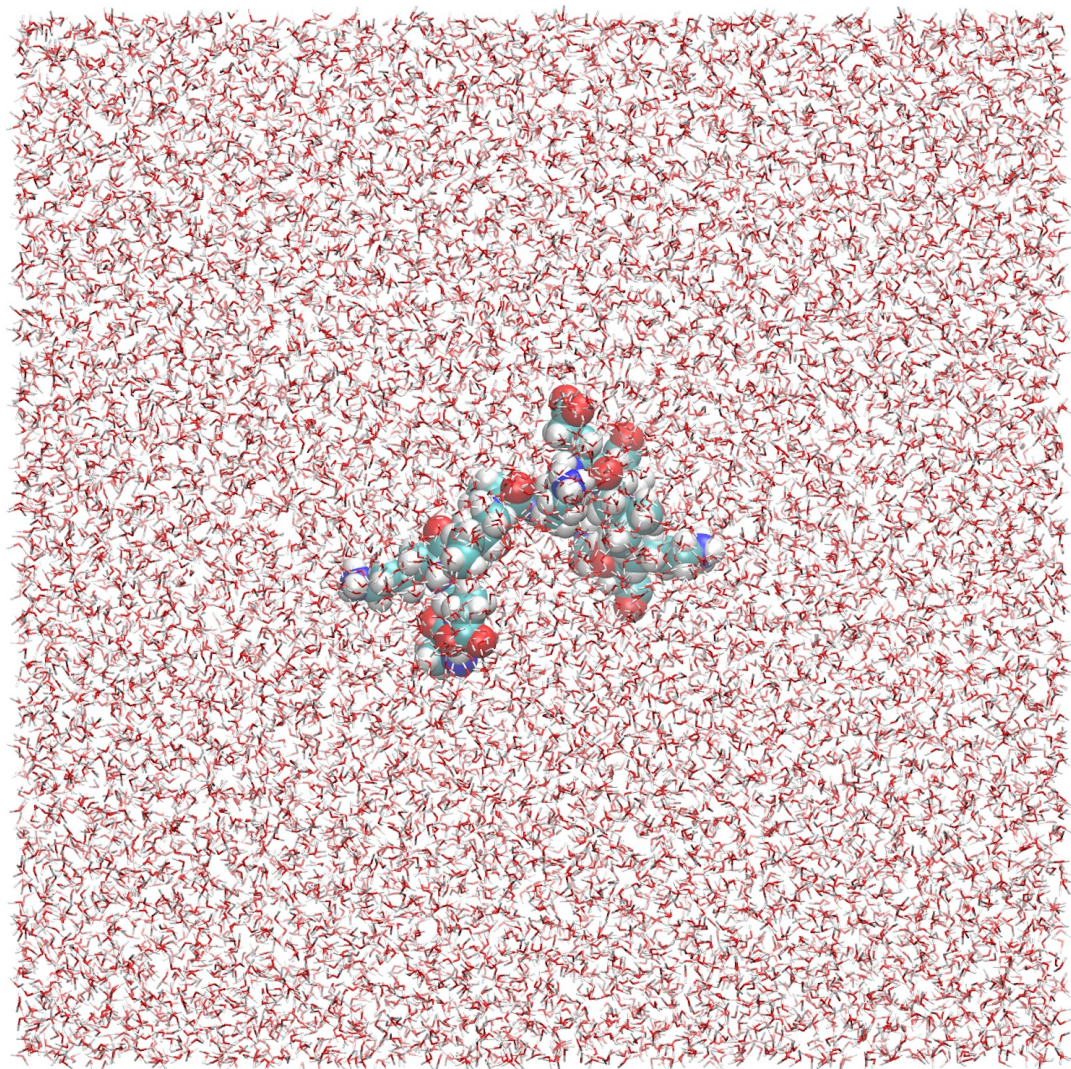


Water density maps

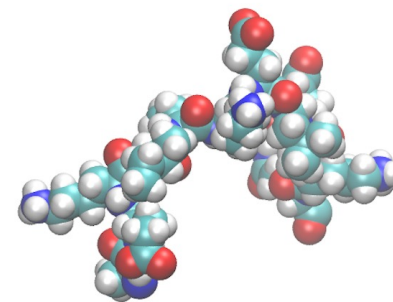
Average number of water molecules in the first two solvation shells

	TIP3P	TIP4P	TIP5P	SPC/E
first shell, <math><3.4 \text{ \AA}</math>	93 ± 8	93 ± 6	100 ± 8	94 ± 7
second shell, <math><5.0 \text{ \AA}</math>	185 ± 14	182 ± 12	196 ± 15	186 ± 13
long-residency water molecules	0.0 ± 0.0	2.0 ± 1.3	12.8 ± 4.5	3.7 ± 2.7

BUT ...



Implicit water as dielectric continuum
can be so much faster



Implicit solvent models

- Represent solvent and counterions as a continuum → solvent degrees of freedom are taken into account implicitly
- Solvent = high dielectric medium & protein = low dielectric region & spatial charge distribution
- No need of water equilibration
- Much faster than explicit solvent → lower computational cost
- Examples
 - Poisson-Boltzmann model
 - Generalized Born model
 - ABSINTH model (explicit ions)

Implicit solvent models

- seek to approximate is the solute potential of mean force, which determines the statistical weight of solute conformations, and which is obtained by averaging over the solvent degrees of freedom.
- the total free energy = the reversible work performed in two successive steps
 - the particle is inserted in the solvent with zero atomic partial charges
 - the atomic partial charges of the particle are switched from zero to their full values
- **the total solvation free energy corresponds to a sum of non-polar and electrostatic contributions**

$$\Delta G_{\text{solv}} = \Delta G_{\text{polar}} + \Delta G_{\text{nonpolar}}$$

Implicit solvent models

$$\Delta G_{\text{solv}} = \Delta G_{\text{polar}} + \Delta G_{\text{nonpolar}}$$

↓
↓

Electrostatics

Poisson-Boltzmann

Born

$\Delta G_{\text{cav}} + \Delta G_{\text{vdW}}$

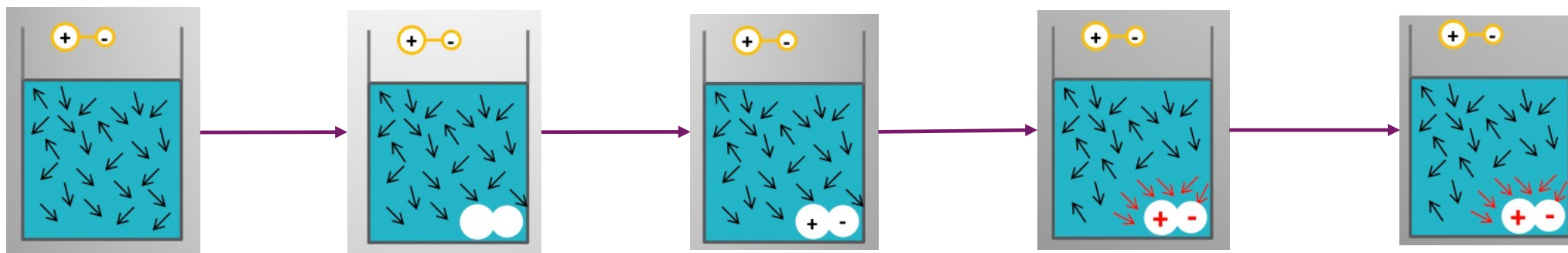
SASA

ΔG_{cav} – cavity creation
within the solvent

ΔG_{vdW} – embedding of
the particle into the cavity

Solute shape cavity of
vacuum is introduced
into the solvent

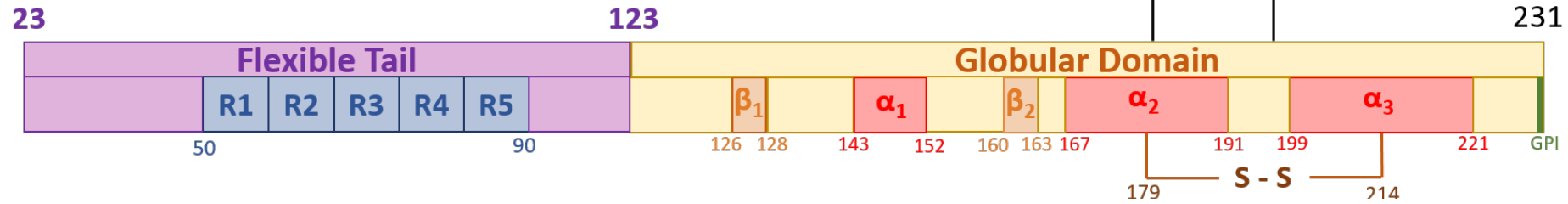
Solvent molecules reorient
and polarize in response to
the solute charge density



Solute charge density is
placed in the solute
cavity

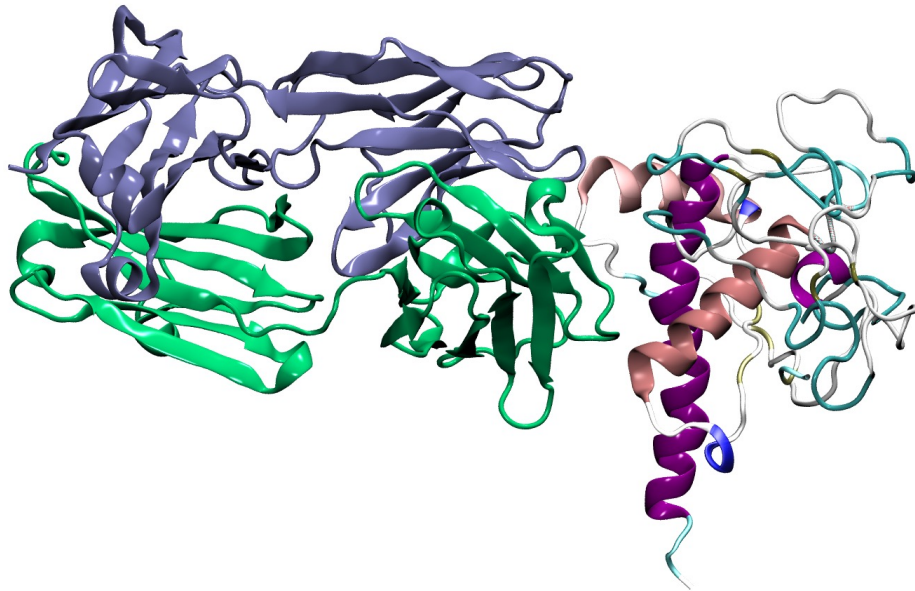
Solute polarizes in response
to solvent polarization

An example



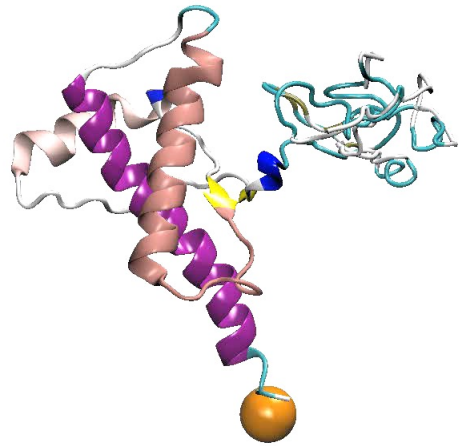
In **explicit solvent** 1.6M atoms
 $\Delta t = 2$ fs

In **implicit solvent**
 $\Delta t = 5$ fs

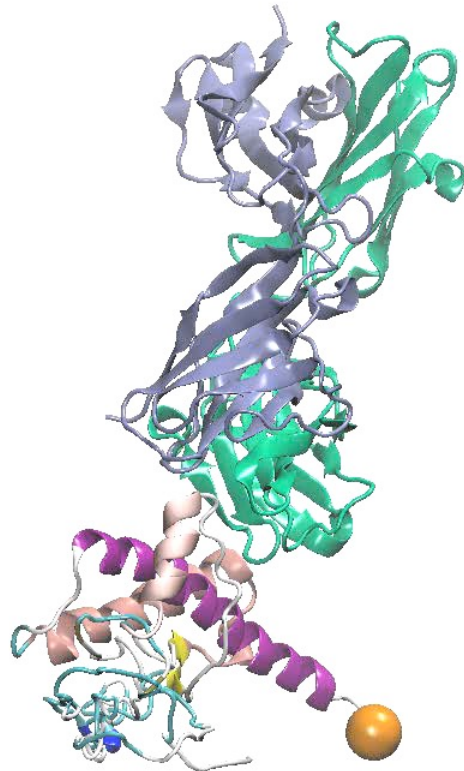


Membrane-interaction mechanisms

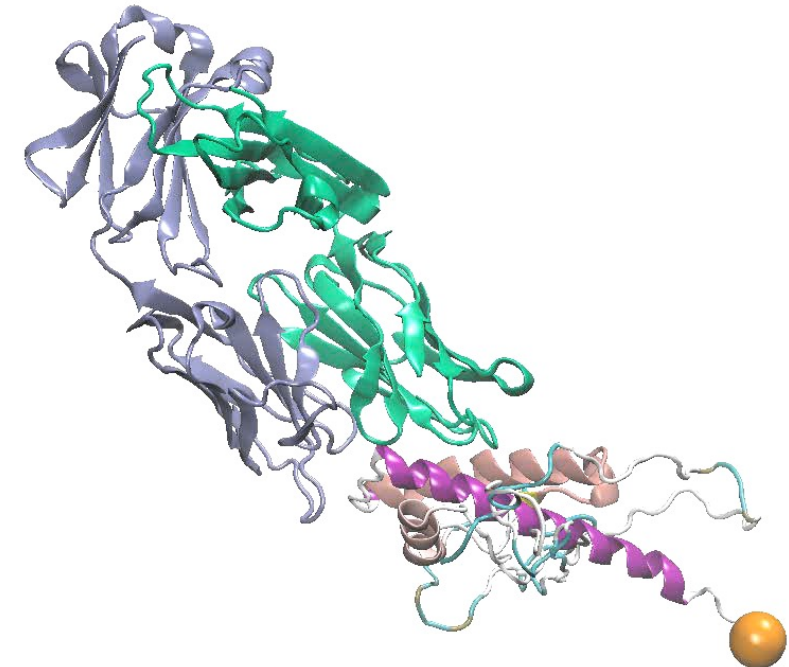
PrP^C



POM1



POM6



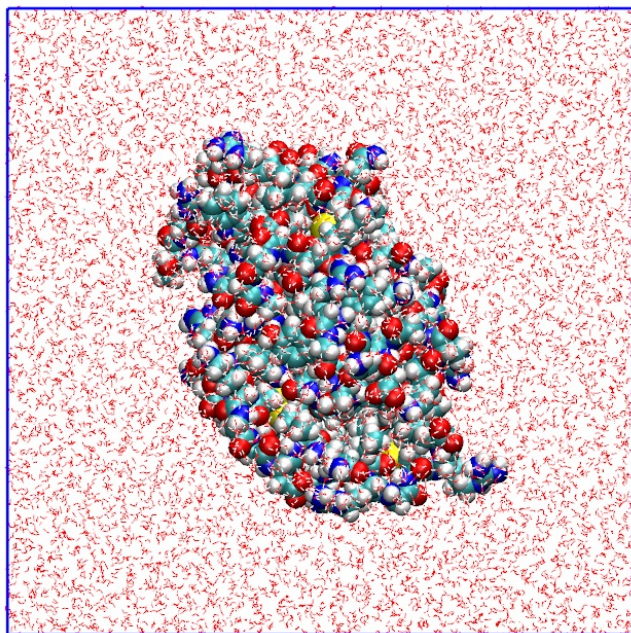
I

1.5

Sp



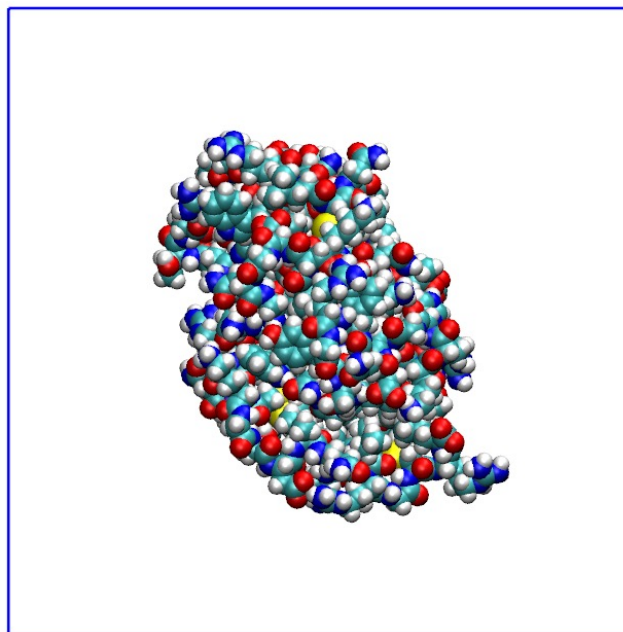
Cows, Langevin & Brown - Can we do more?



Solvent molecules

$\langle \text{collisions solvent} \rangle$
 fluctuations around ξ

(thermal noise of solvent molecules)



stochastic force



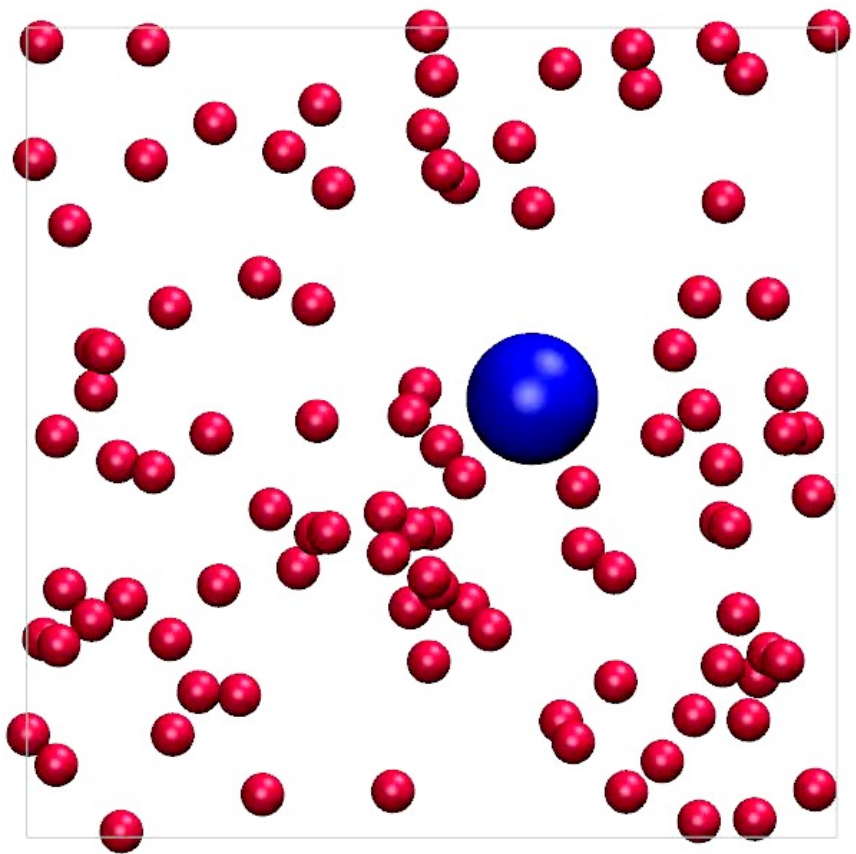
friction, ξ



stochastic Markovian process



Langevin & Brown



$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt}$$

$$m \frac{d\boldsymbol{v}}{dt} = -\boldsymbol{F}_D + \boldsymbol{F}_B$$

From hydrodynamics $\boldsymbol{F}_D = 6\pi\eta\boldsymbol{v}r$

$$m \frac{d\boldsymbol{v}}{dt} = -6\pi\eta\boldsymbol{v}r + \boldsymbol{F}_B$$

Friction force

Random force

$$\xi = 6\pi\eta r / m$$

Langevin dynamics

$$m \frac{d\mathbf{v}}{dt} = -6\pi\eta\mathbf{v}r + \mathbf{F}_B$$



$$\mathbf{v}(t) = \mathbf{v}_0 e^{-\xi t/m} + \int_0^t d\tau e^{-\xi(t-\tau)/m} \frac{1}{m} \mathbf{F}_B(\tau)$$

$$\mathbf{v}^2(t \rightarrow \infty) \approx 0 \text{ false}$$



At **EQUILIBRIUM**: $\langle \mathbf{v}^2 \rangle_{eq} = \frac{kT}{m}$

$$\langle \mathbf{F}_B(t) \rangle = 0 \quad \text{Noise produces no net force}$$

$$\langle \mathbf{F}_B(t) \cdot \mathbf{F}_B(t') \rangle = 2k_B T \xi \delta(t - t')$$

Fluctuation-dissipation theorem

Balance between “dead” and “alive”

δ -Dirac function

No correlation between t and t'
 due to time scale separation
 (short lived collisions)

Langevin dynamics

$$m\ddot{\mathbf{r}} = -\xi\dot{\mathbf{r}} + \mathbf{F}(\mathbf{r}) + \sigma\mathbf{R}$$

$$\sigma = 2k_B T \xi$$

Reduced degrees of freedom

Focus on the important contributions



Keeps the contributions to the dynamics of the system



Long chain molecules



Barrier crossing motions

Brownian dynamics

= overdamped Langevin ($\mathbf{a}=0$)

- No inertia (high friction)

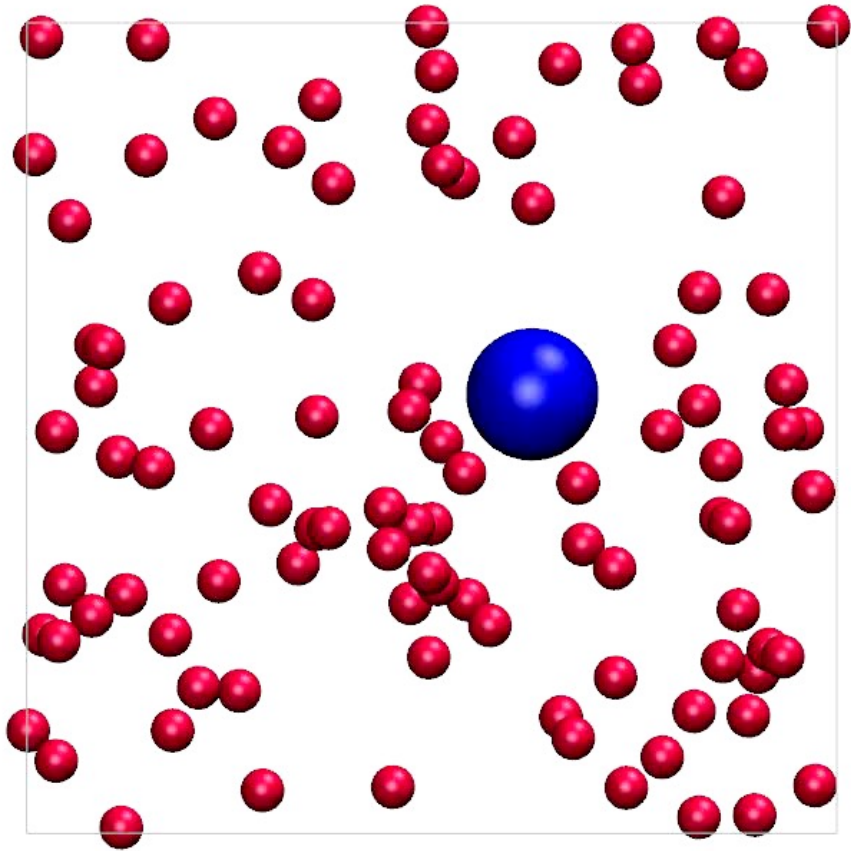
$$m\ddot{\mathbf{r}} = -\xi\dot{\mathbf{r}} + \mathbf{F}(\mathbf{r}) + \sigma\mathbf{R}$$

$$\langle m\ddot{\mathbf{r}} \rangle_{\Delta t} \cong 0$$

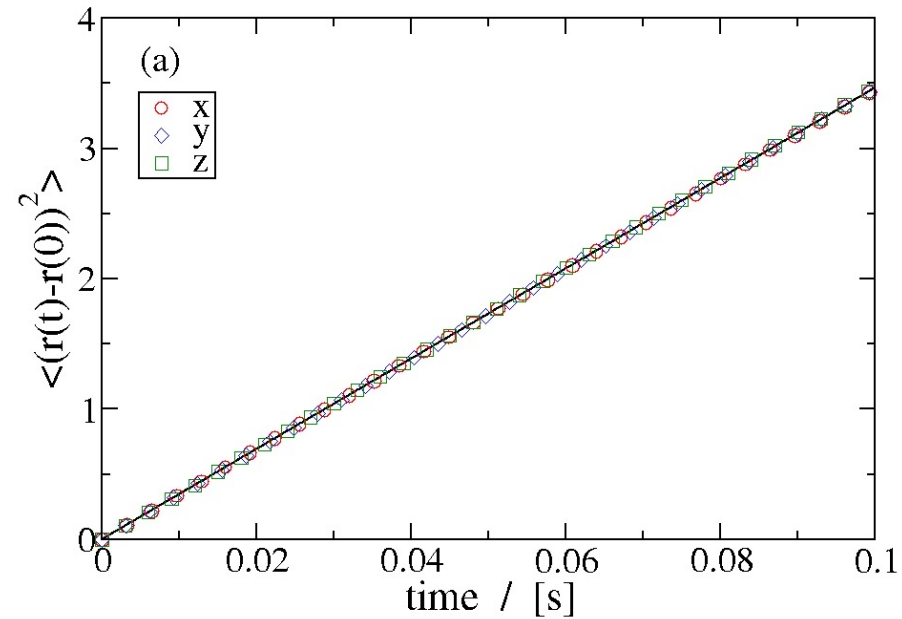
$$\sigma = 2k_B T \xi$$

$$\Delta\mathbf{r} = \frac{\mathbf{F}}{\xi} + \sqrt{\frac{2k_B T \Delta t}{\xi}} \mathbf{R}$$

In practice



$$\mathbf{r}(t + \delta t) - \mathbf{r}(t) = \boldsymbol{\mu}^t \mathbf{F} \delta t + \sqrt{\boldsymbol{\mu}^t \boldsymbol{\Theta}^t(t)} \sqrt{2k_B T \delta t}$$

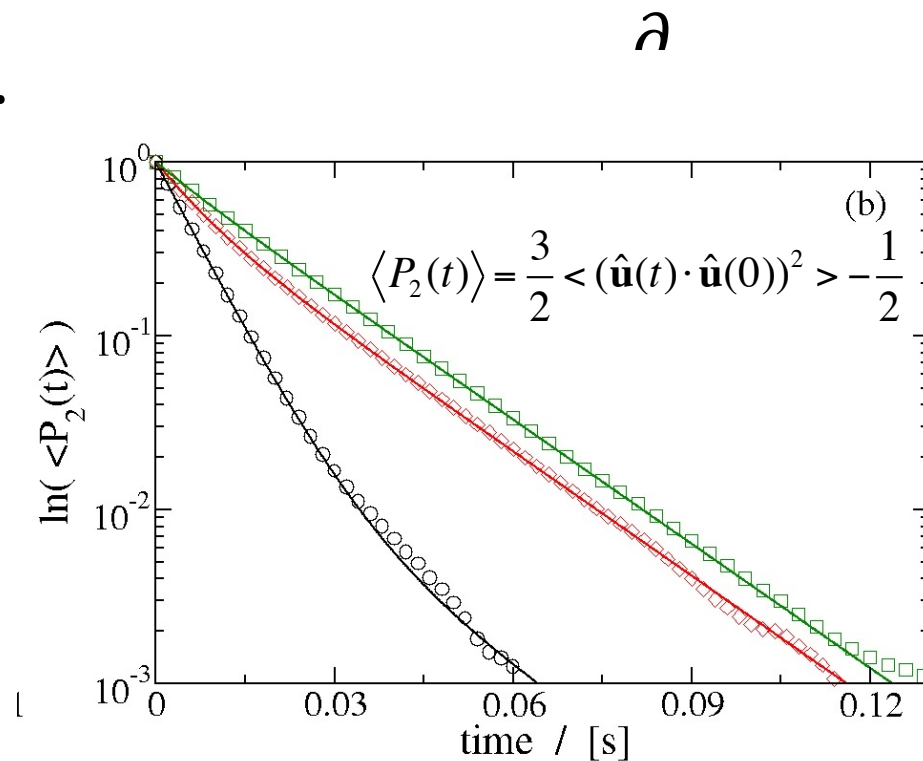


$$\Theta_{\alpha}^i \begin{cases} \langle \Theta_{\alpha}^i(t) \rangle = 0 \\ \langle \Theta_{\alpha}^i(t) \Theta_{\beta}^j(t') \rangle = \delta_{\alpha\beta} \delta_{ij} \delta_{tt'} \end{cases}$$

$$\langle (\mathbf{r}(t) - \mathbf{r}(0))^2 \rangle = 6D_t \delta t$$

In practice

$$\mathbf{q}(t + \delta t) - \mathbf{q}(t) = \mathbf{A} \cdot$$



$$\sqrt{\boldsymbol{\mu}^r \boldsymbol{\Theta}^r(t)} \sqrt{2k_B T \delta t}$$

↓
rotational friction

$$\mathbf{q}(t + \delta t) - \mathbf{q}(t) = \mathbf{B} \cdot \boldsymbol{\mu}^r \cdot \mathbf{A}^T \mathbf{T} \delta t + \mathbf{B} \sqrt{\boldsymbol{\mu}^r \boldsymbol{\Theta}^r(t)} \sqrt{2k_B T \delta t} + \lambda \mathbf{q}$$

$$q(t + \delta t) = 1$$

$$\lambda^2 + 2\lambda \mathbf{q}(t) \cdot \tilde{\mathbf{q}}(t + dt) + \tilde{\mathbf{q}}^2(t + \delta t) = 1$$

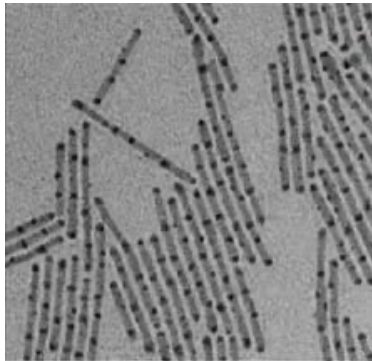
Intermezzo – the ideal gas and beyond



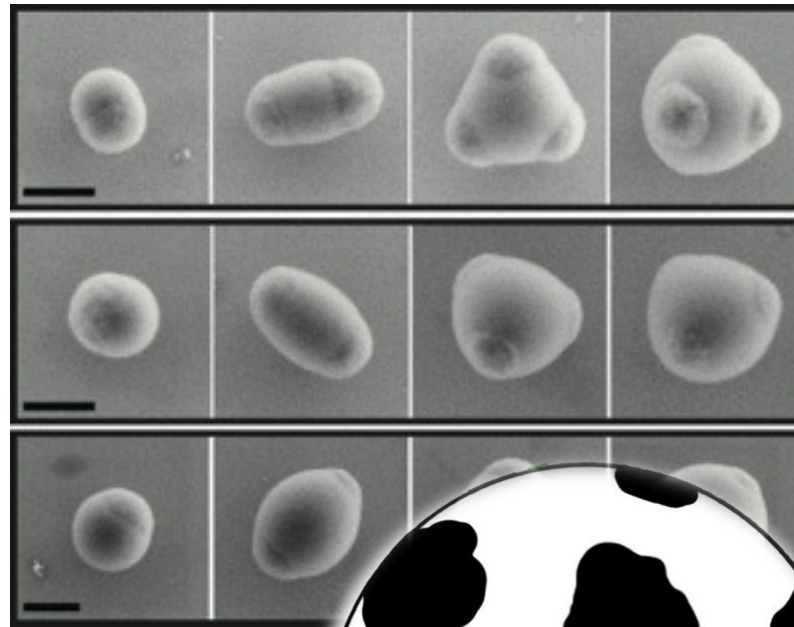
Janus particles



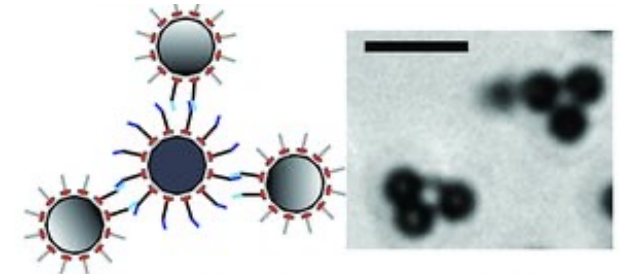
Striped nanospheres



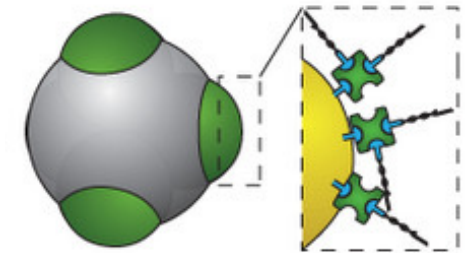
Striped nanorods



Wang ... Pi

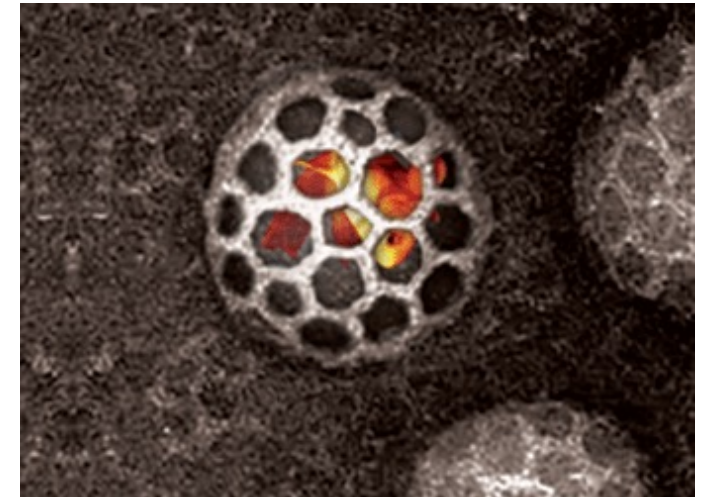
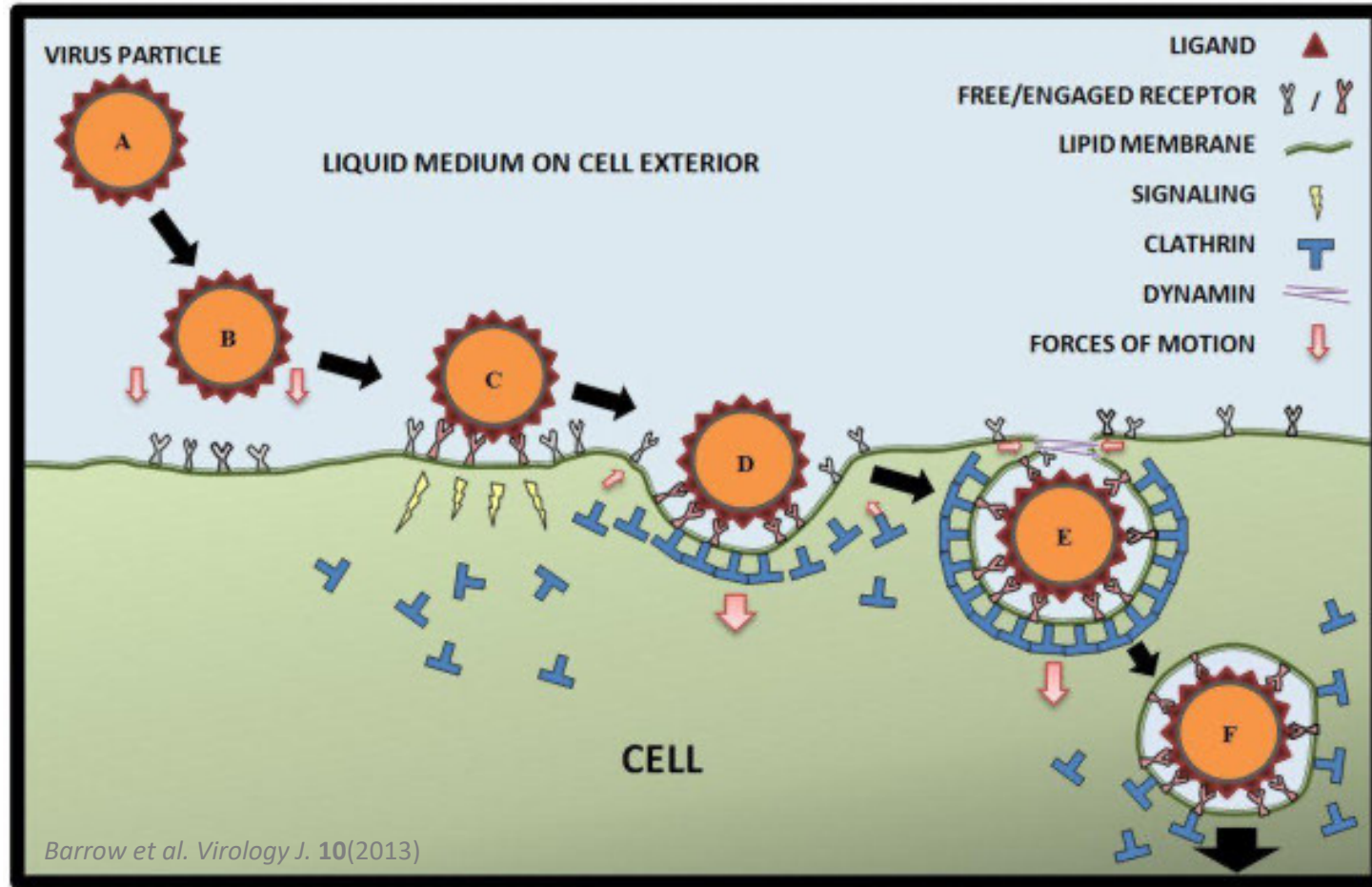


DNA patchy particles



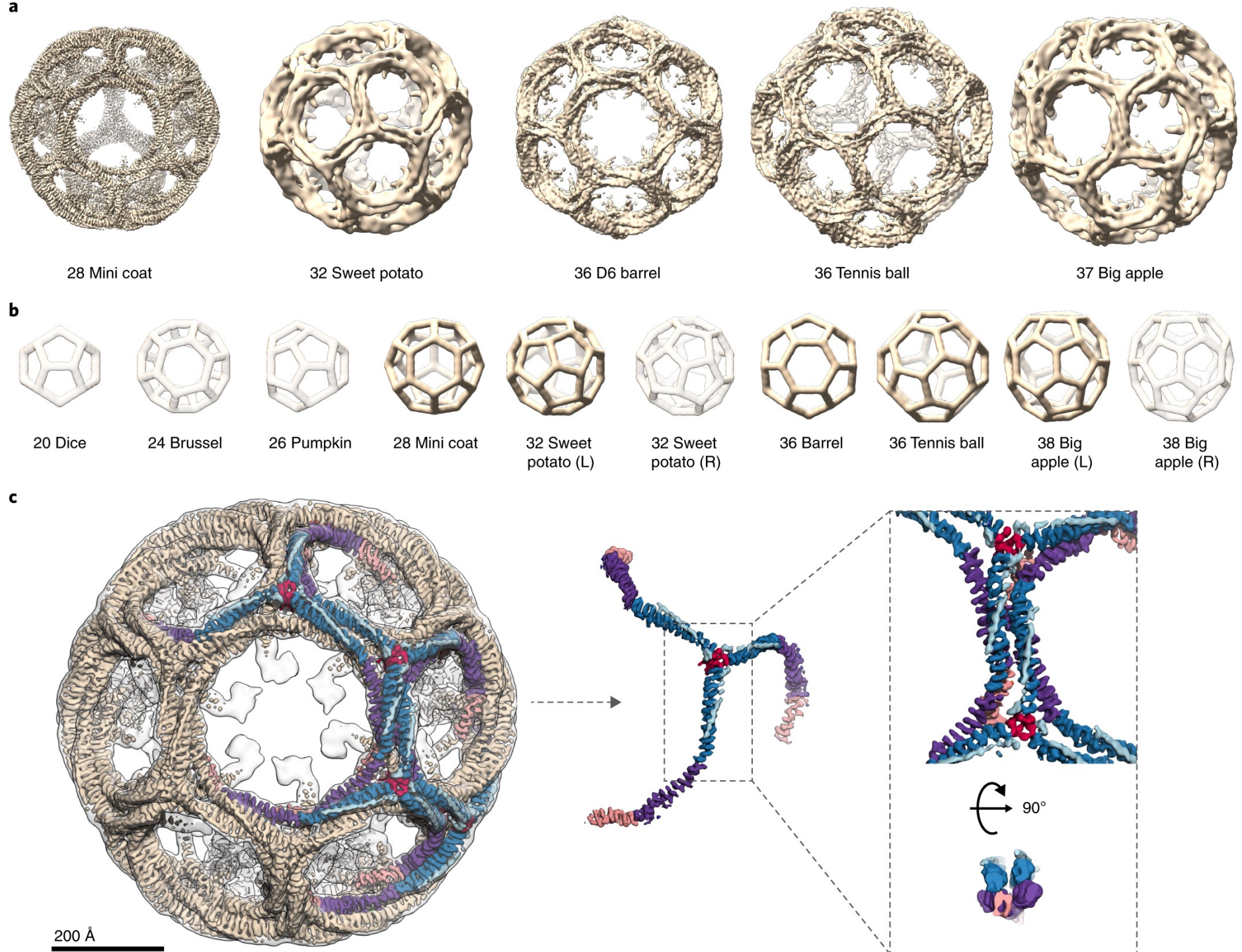
DNA
patches

An example – proteins as patchy particles ☺

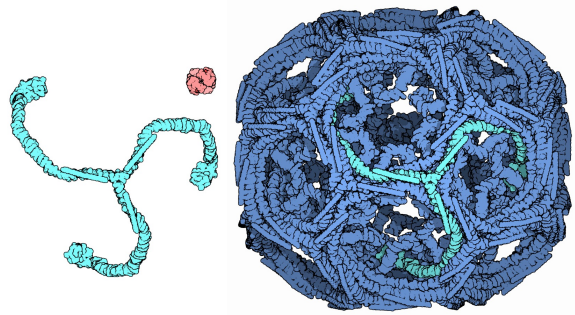


www.endocytosis.org

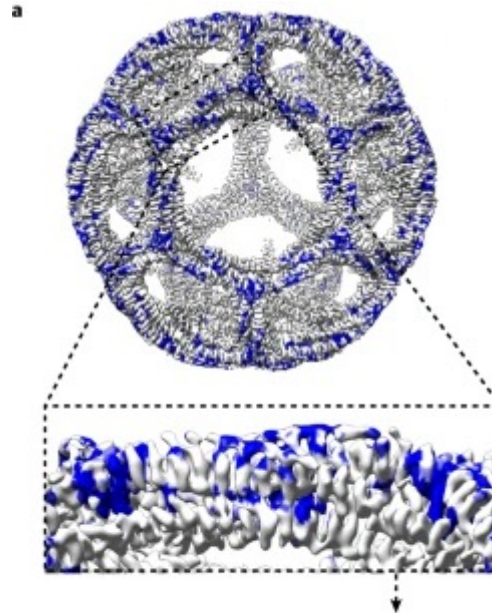
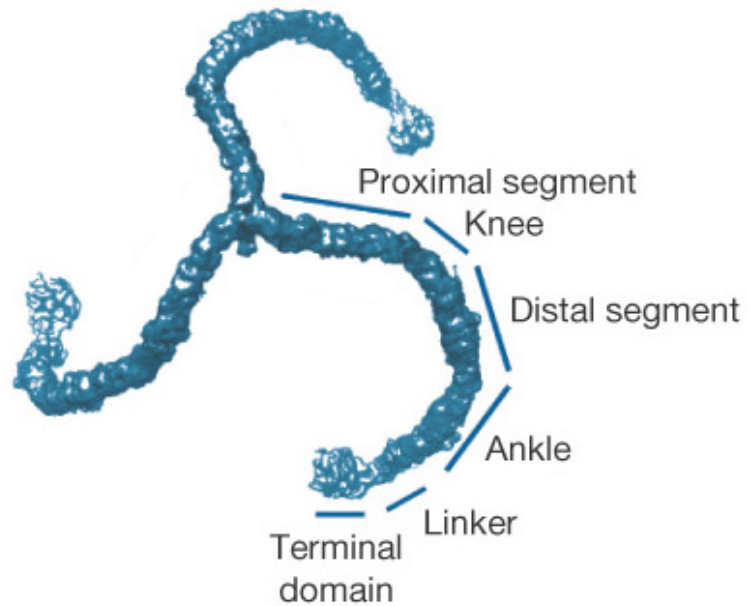
The model clathrin



The model

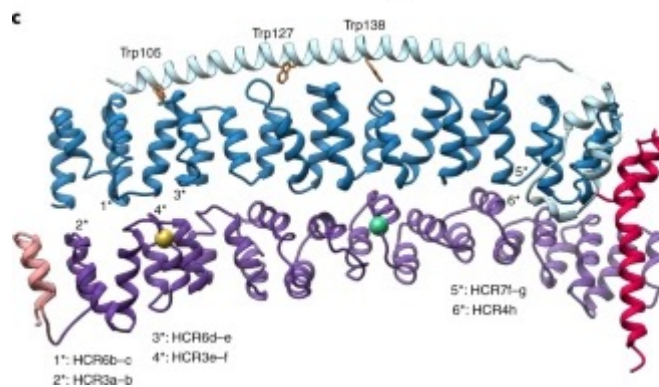


G. T. Johnson & D. Goodsell, RCSB PDB(2007)

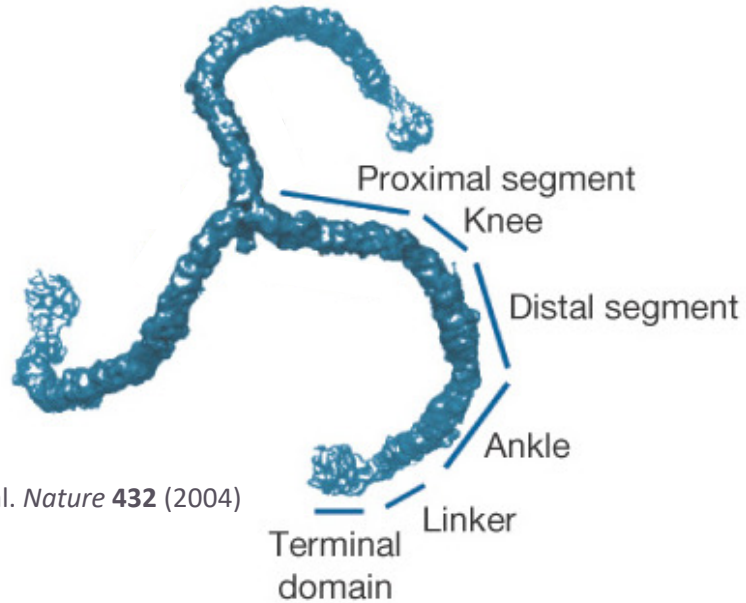


5025 AA → 645 kDa
 a PIECE!!!!

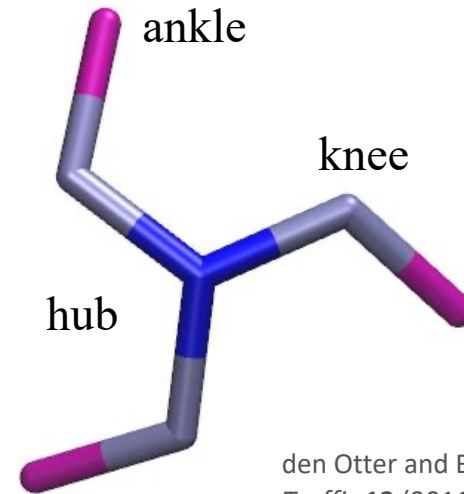
Atomistically impossible
 → coarse graining



Coarse-grained model: one protein = 1 rigid particle

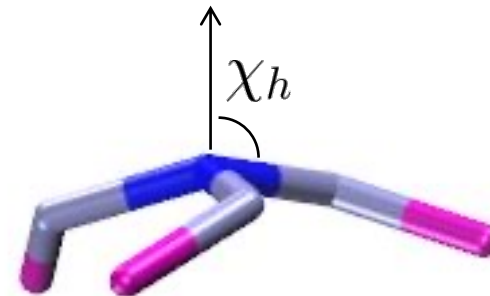


A. Fotin et al. *Nature* **432** (2004)



den Otter and Briels,
Traffic **12** (2011)

- Slightly curved structure

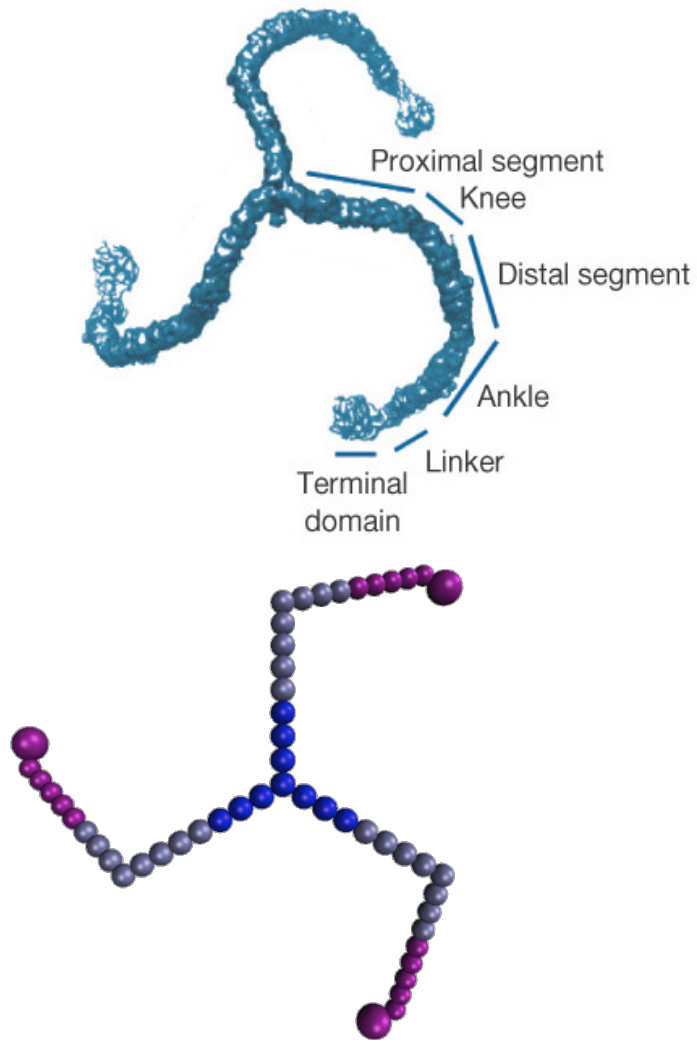


What do we need?

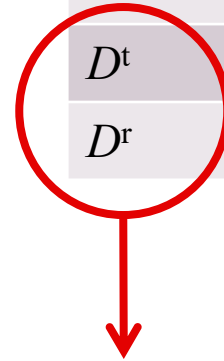
- Friction
- Interaction potential
- A good cluster

$$\mathbf{r}(t + \delta t) - \mathbf{r}(t) = \mathbf{A} \cdot \boldsymbol{\mu}^t \cdot \mathbf{A}^T \mathbf{F} \delta t + \mathbf{A} \cdot \sqrt{\boldsymbol{\mu}^t \boldsymbol{\Theta}^t(t)} \sqrt{2k_B T \delta t}$$

Diffusion



HYDRO++*	
Input	
M.W.	645 kDa
η	0.01 Poise
χ	101°
T	20 °C
Output	
D^t	$1.22 \cdot 10^{-7} \text{ cm}^2/\text{s}^{**}$
D^r	$1.43 \cdot 10^4 \text{ s}^{-1}$



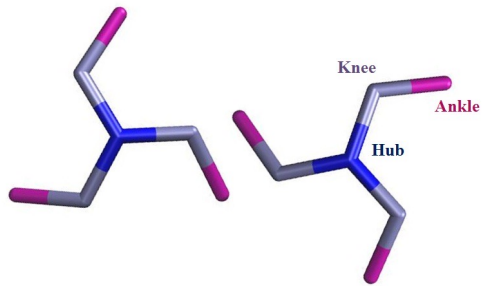
Diffusion matrices $D = \frac{k_B T}{\xi}$

*J. García de la Torre, D. Amorós, A. Ortega, *Eur. Biophys. J.* **39**, 381-388 (2010).

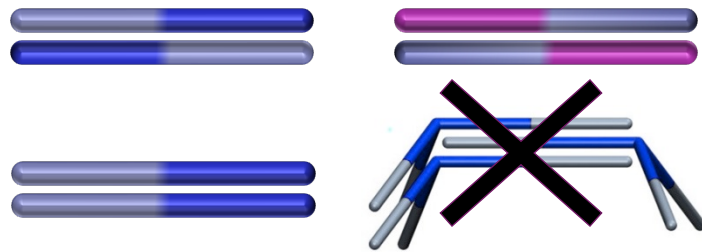
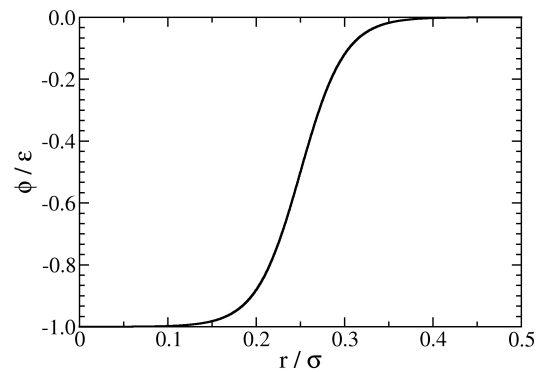
** T. Yoshimura, K. Kameyana, S. Maezwa and T. Takagio, *Biochemistry* **20**, 4528 (1991).

The potential

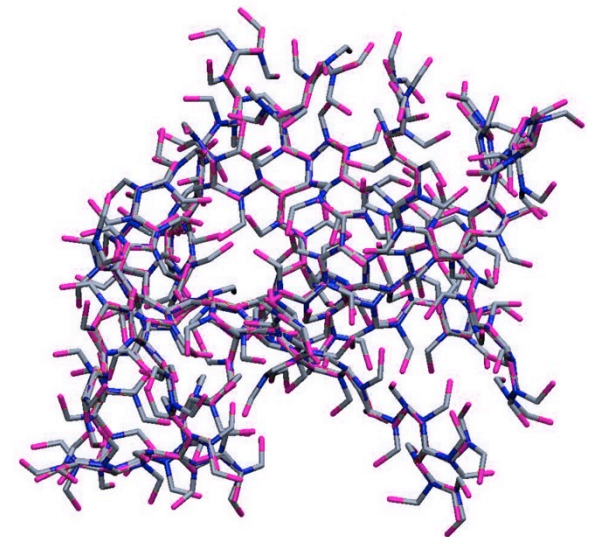
- interaction between leg segments



$$\Phi = -\varepsilon \cdot f(r)$$



- Attraction between anti-parallel legs
- Repulsion between parallel legs

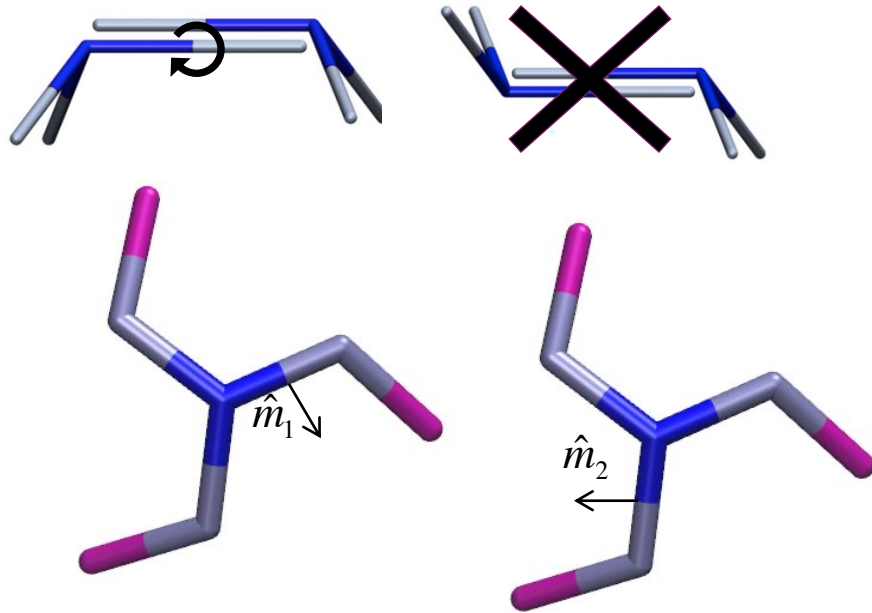


den Otter et al, *Biophys. J.* **99** (2010)

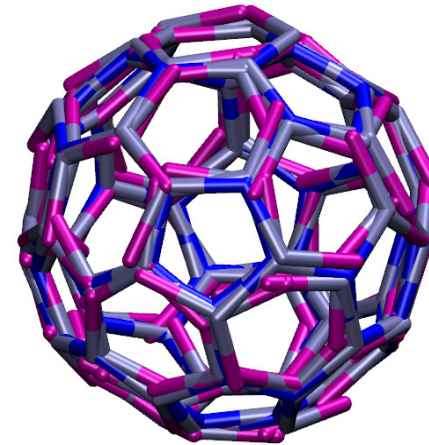
The potential

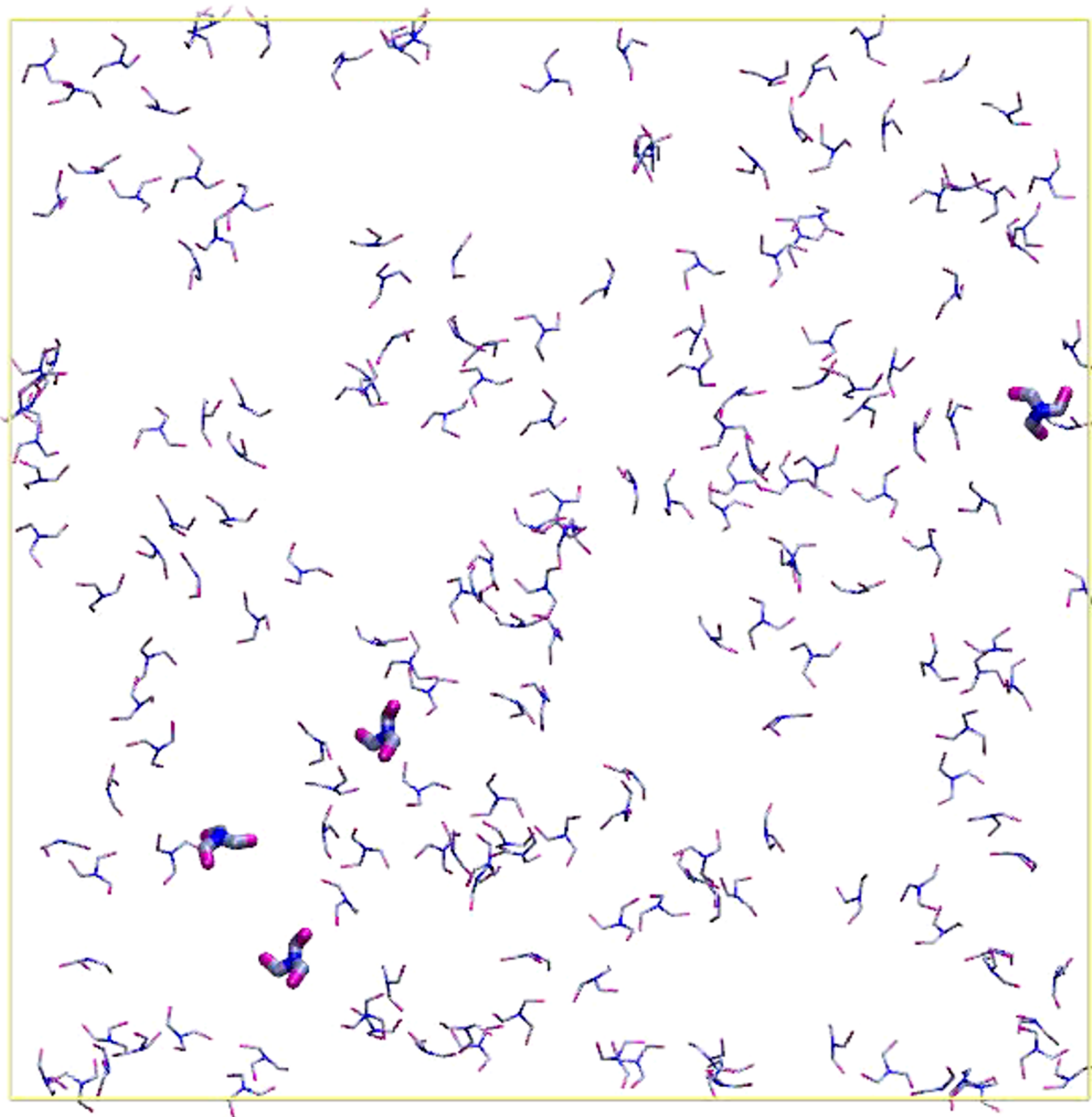
- interaction between leg segments
- asymmetric binding

$$\Phi = -\varepsilon \cdot f(r)$$

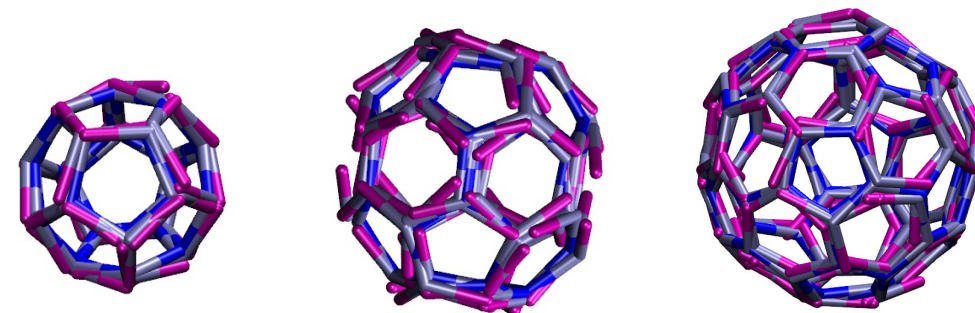
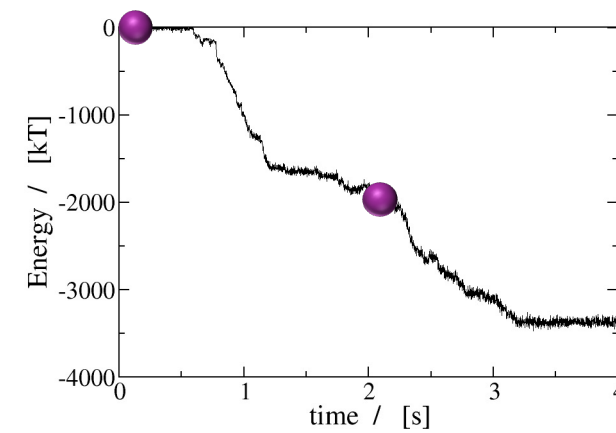


- Rotational asymmetry





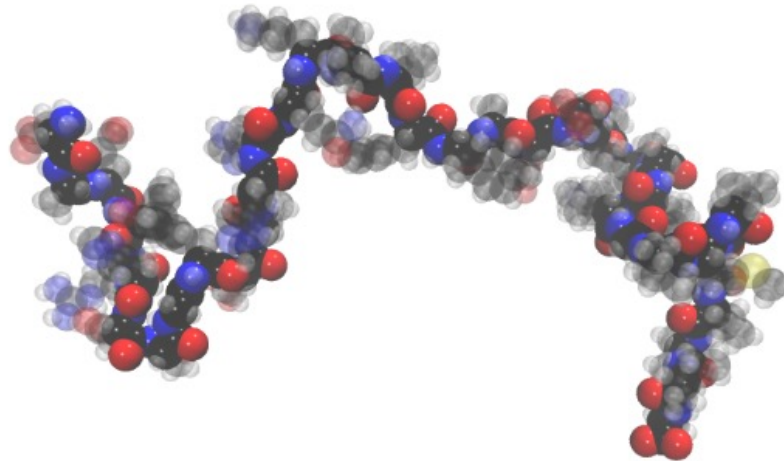
N particles	200
c	100 $\mu\text{g/ml}$
T	Temp = 20 $^{\circ}\text{C}$
δt	10 ns
timescale	4 s
box length	1 μm



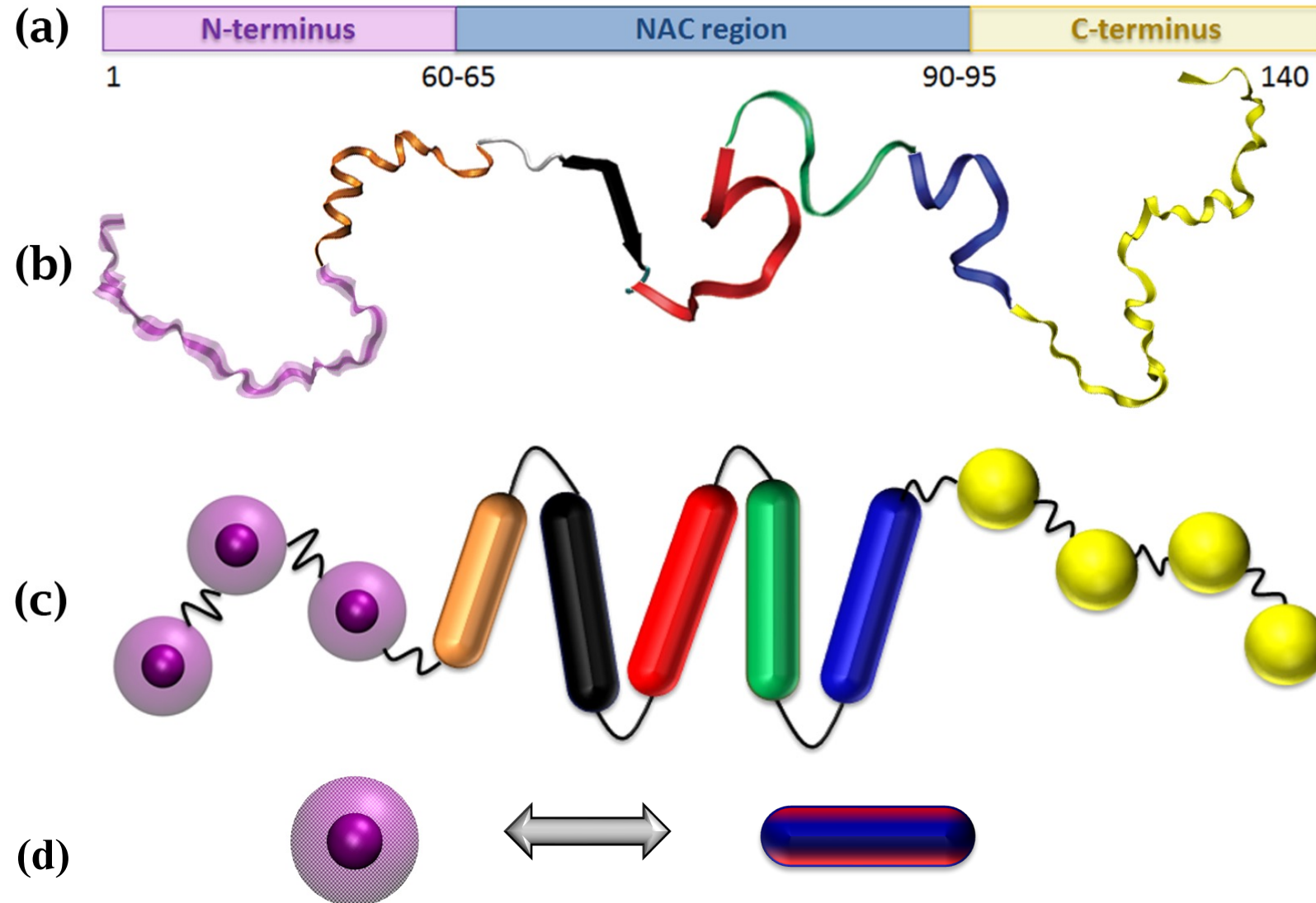
Another example – a sophisticated protein

IDP = intrinsically disordered = little or no structure in its native state

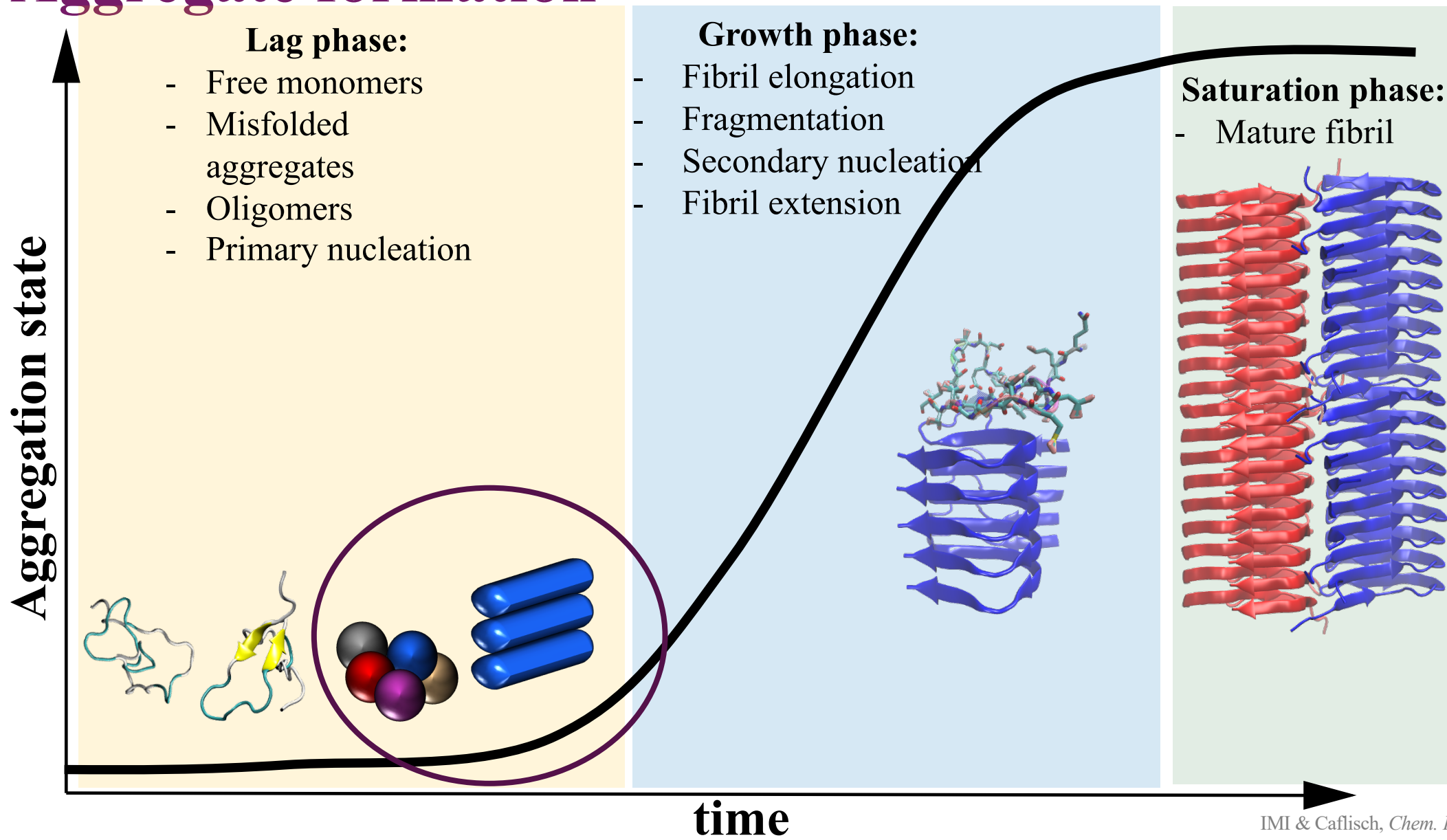
1	60-65	90-95	140
N-terminus	NAC region	C-terminus	
<ul style="list-style-type: none"> - amphipathic (both hydrophilic and lipophilic) - α-helix, disordered 	<ul style="list-style-type: none"> - hydrophobic - building block for α-synuclein aggregates - α-helix, β-sheets, disordered 	<ul style="list-style-type: none"> - highly acidic - negatively charged - disordered 	



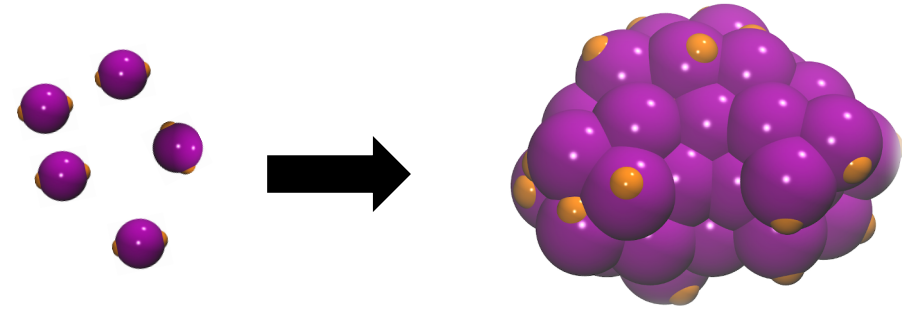
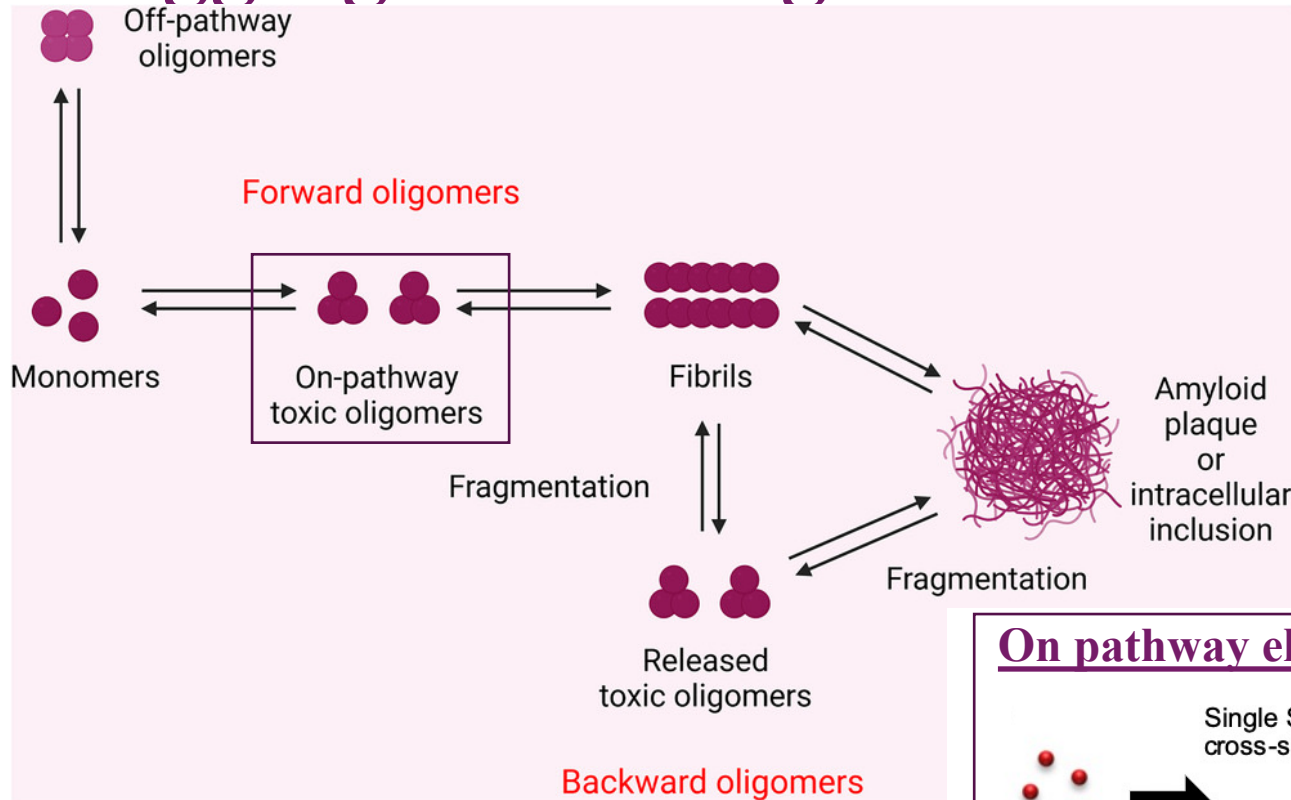
How to CG? What is my research question?



Aggregate formation

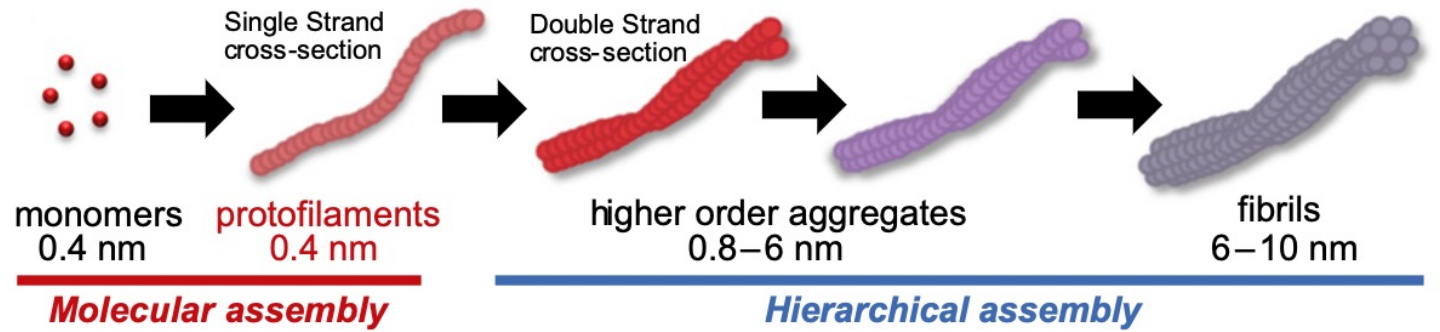


Aggregates = Oligomers



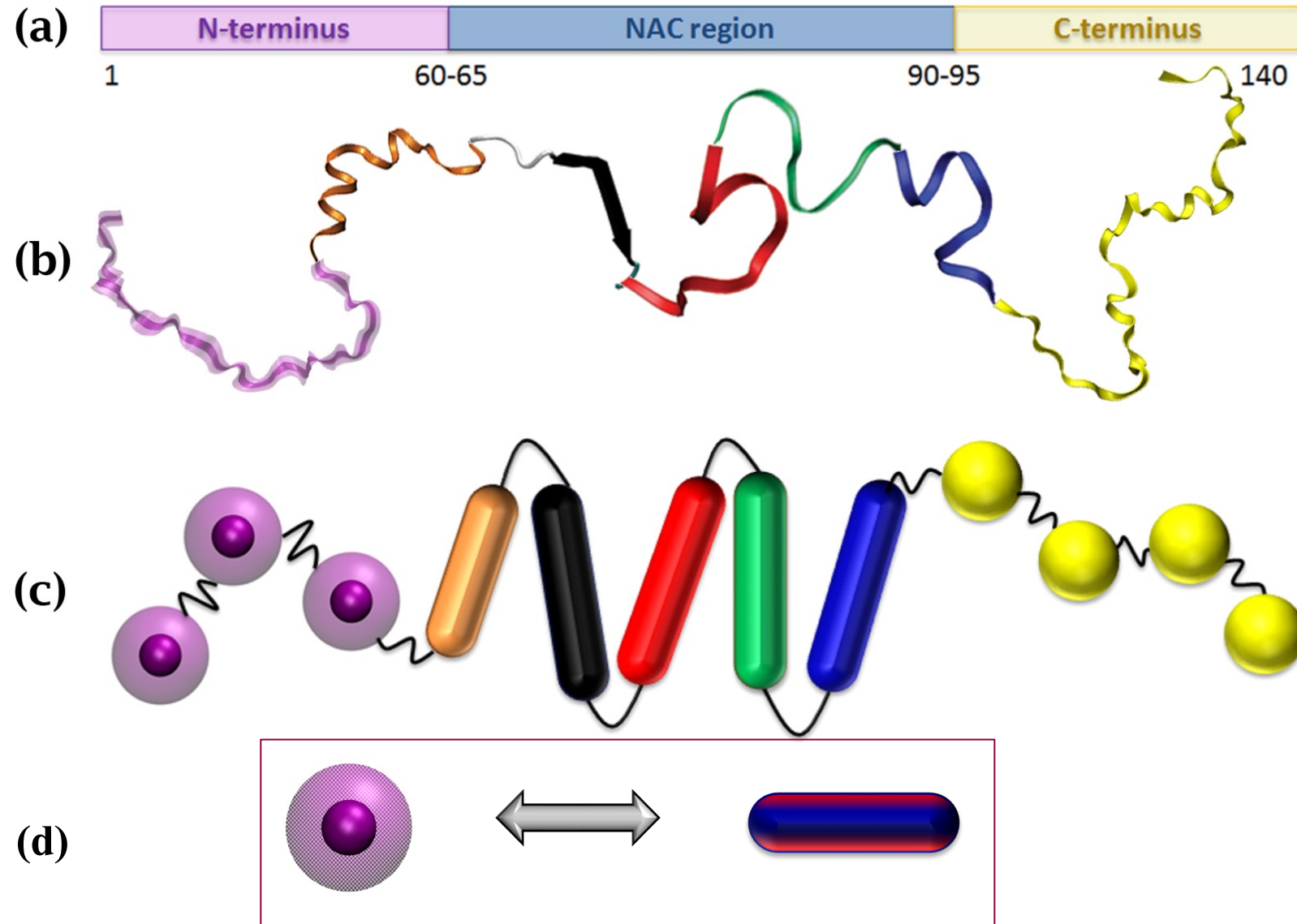
On pathway elongated oligomers = protofibrils

Ruggeri et al. *PNAS*, 2018, 115, 7231



What are the aggregation mechanisms?

What is my research question? Aggregation



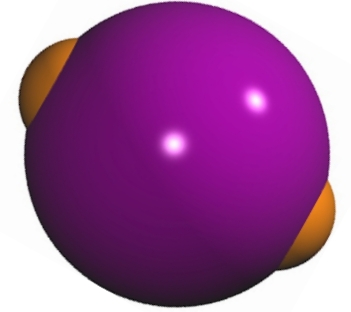
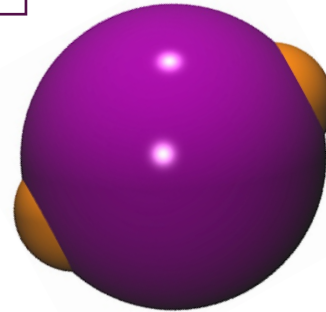
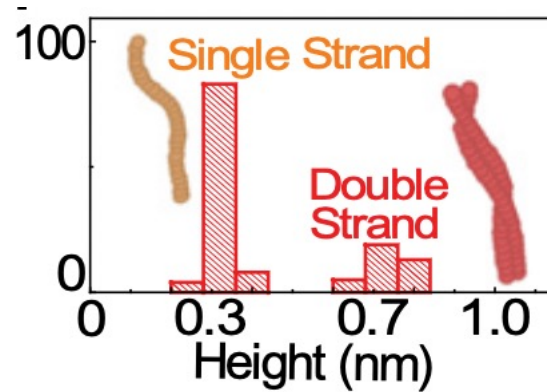
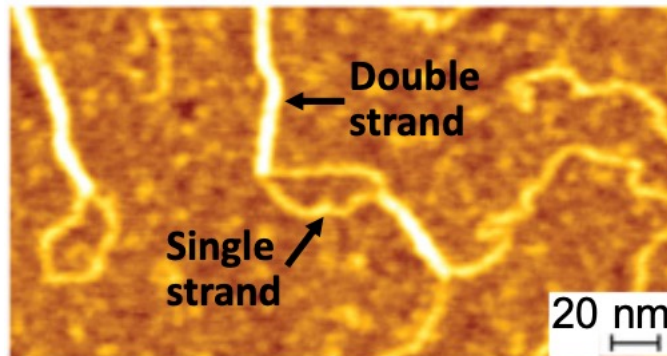
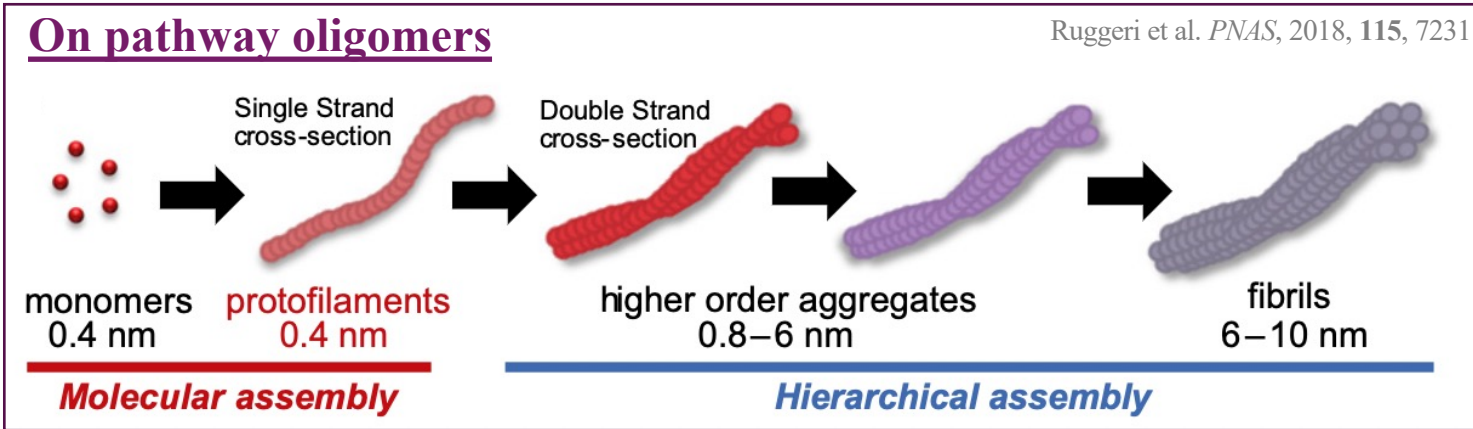


Dr. Simone Ruggeri



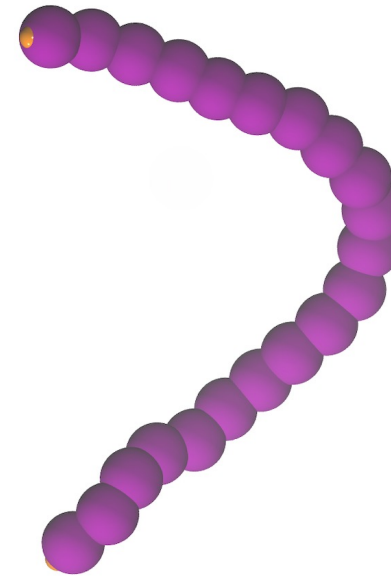
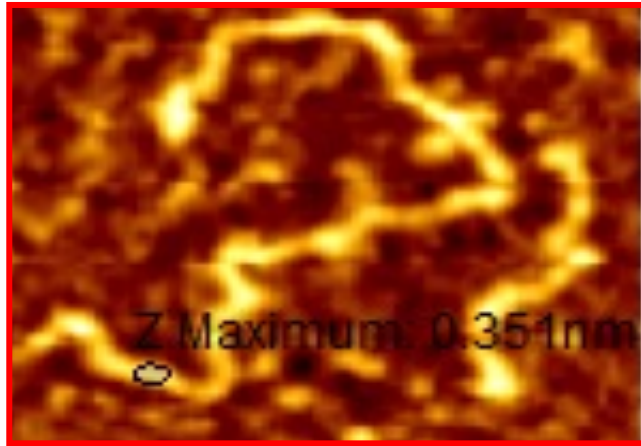
Bernadette Mayer

Aggregates = oligomers



$$\Phi = \sum_{i,j} \Phi_{ex.v}(r_{ij}) + \sum_{i,j} \Phi_{attr}(r_{ij}) + \sum_{i,j} \Phi_{attr}(d_{ij}, \hat{\mathbf{u}}_i, \hat{\mathbf{u}}_j)$$

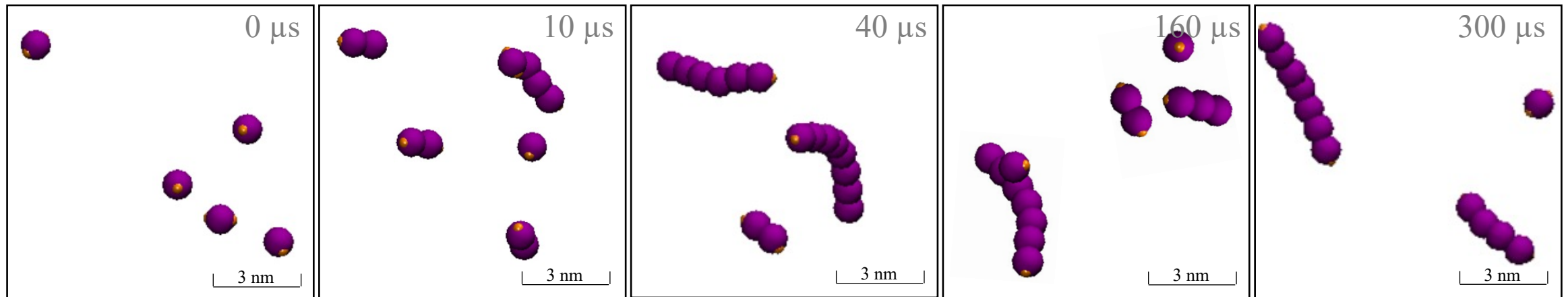
Single-strand protofilaments



Dr. Simone Ruggeri

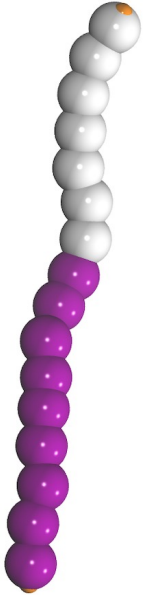


Bernadette Mayer

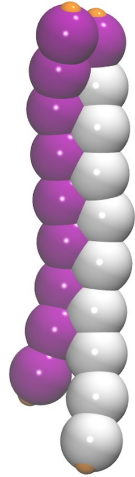


Single-strand mechanisms

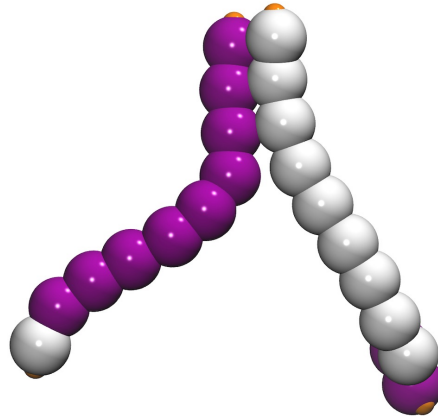
Monomer addition



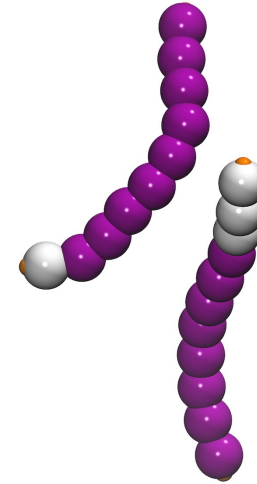
break & self-fold



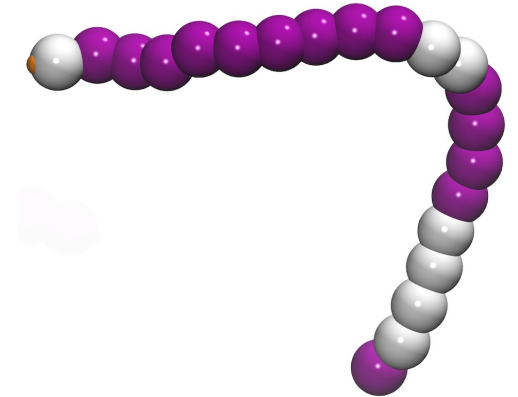
separate



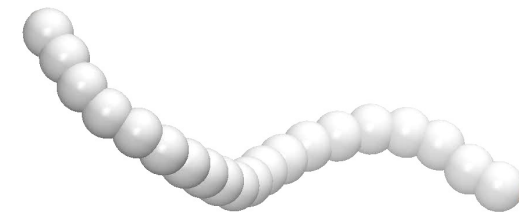
divide



merge



- Driven by specific and non-specific interactions
- The protofilaments are highly dynamic & flexible



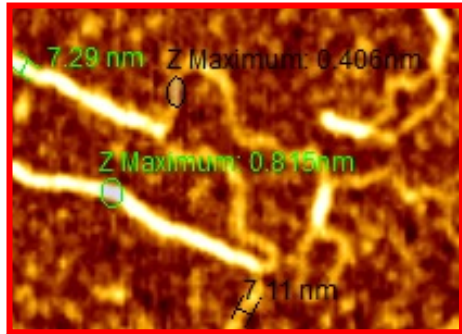


Dr. Simone Ruggeri

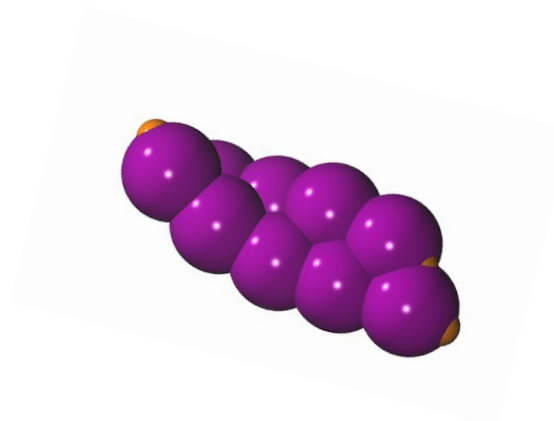


Bernadette Mayer

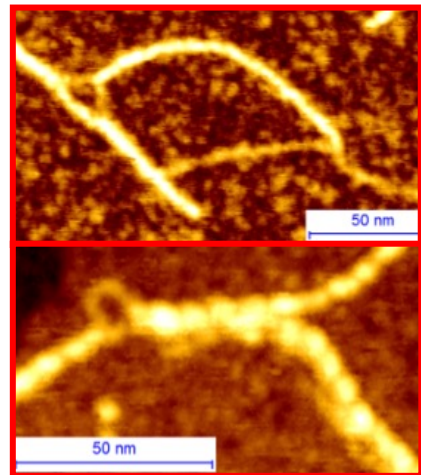
Multi-strand protofilaments



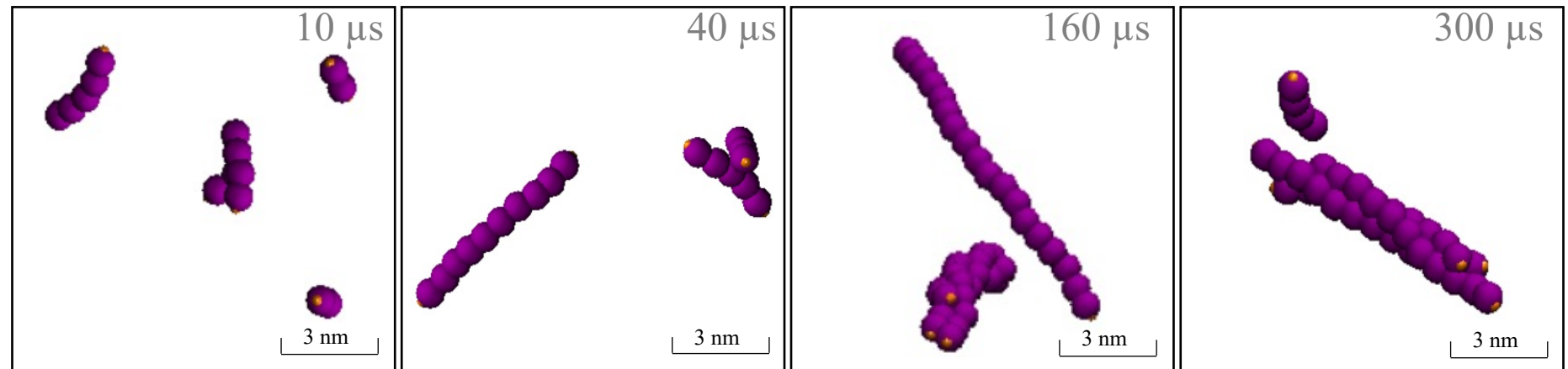
Double strand
protofilaments



- short
- transient species – *on-pathway*
- more rigid than single protofilament precursors



Higher order joining
protofilaments

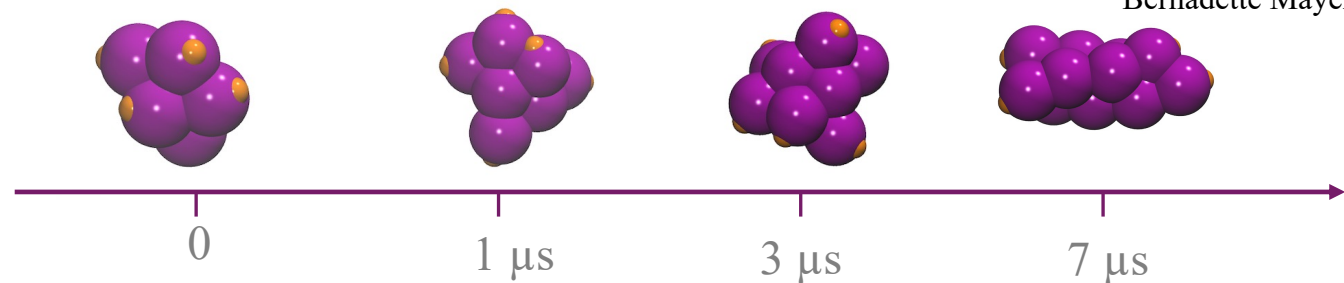


Multi-strand mechanisms

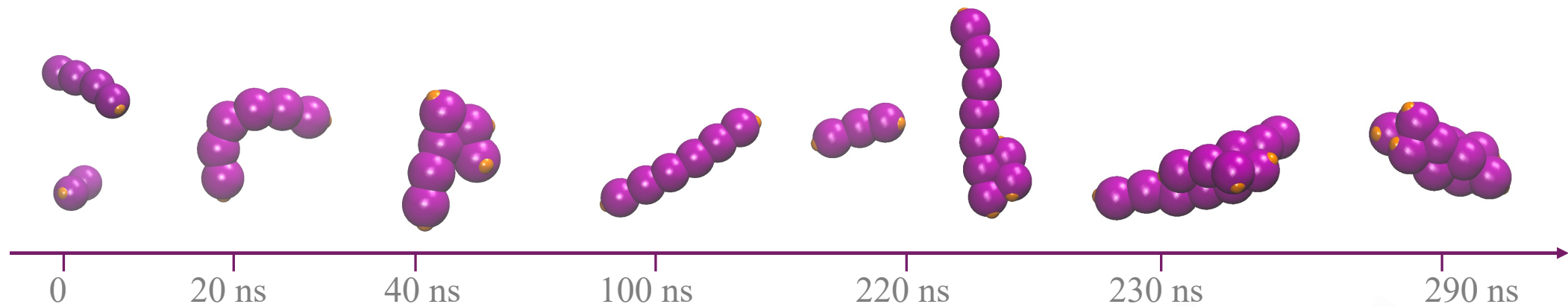


Bernadette Mayer

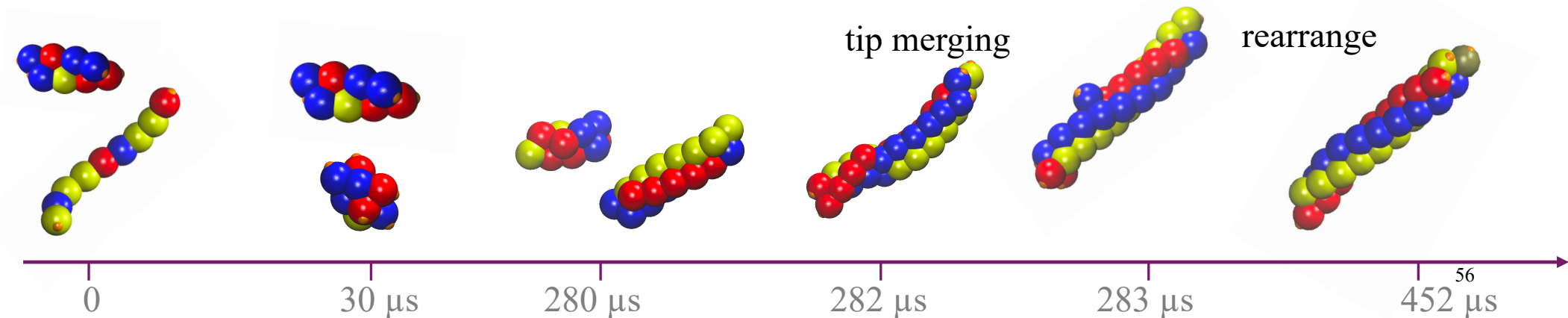
- via spherical oligomers
- driven by non-specific interactions
- stabilized by specific interactions



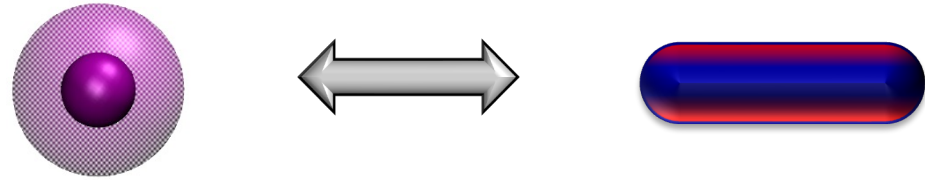
- via breakage of single filaments
- rearrangements



- via tip merging of species
- long rearrangements

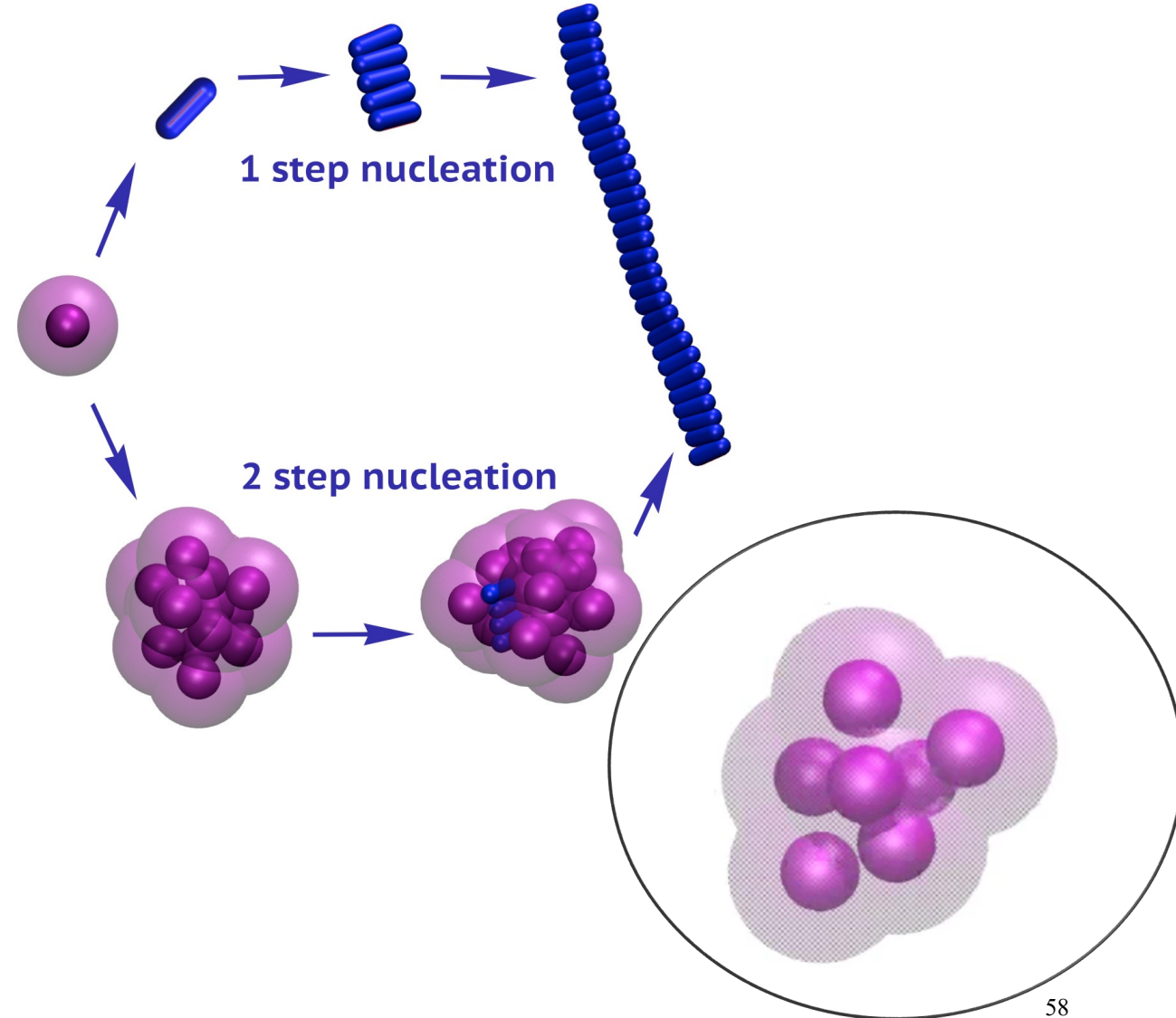
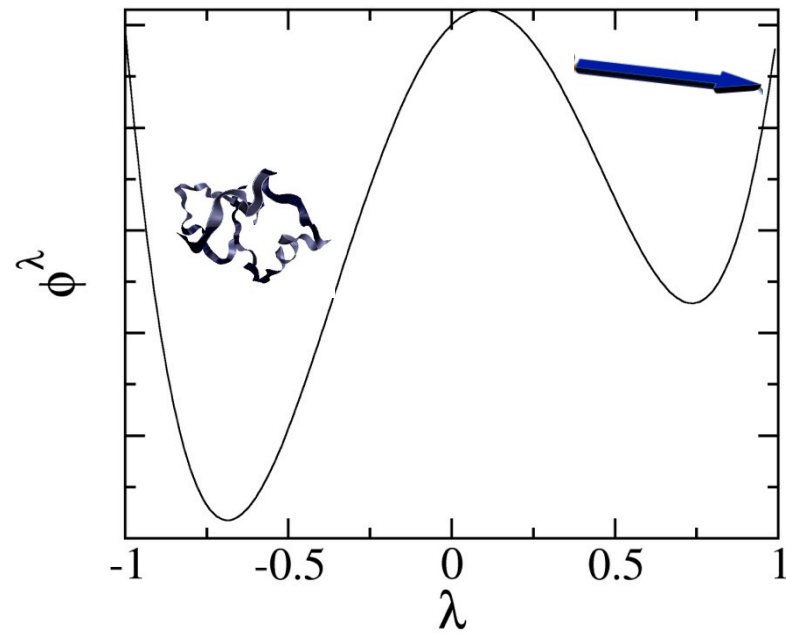


Spherical oligomers & fibrils



- free
- favored internally

- bonded
- internal + inter-particle



Different type of coarse graining

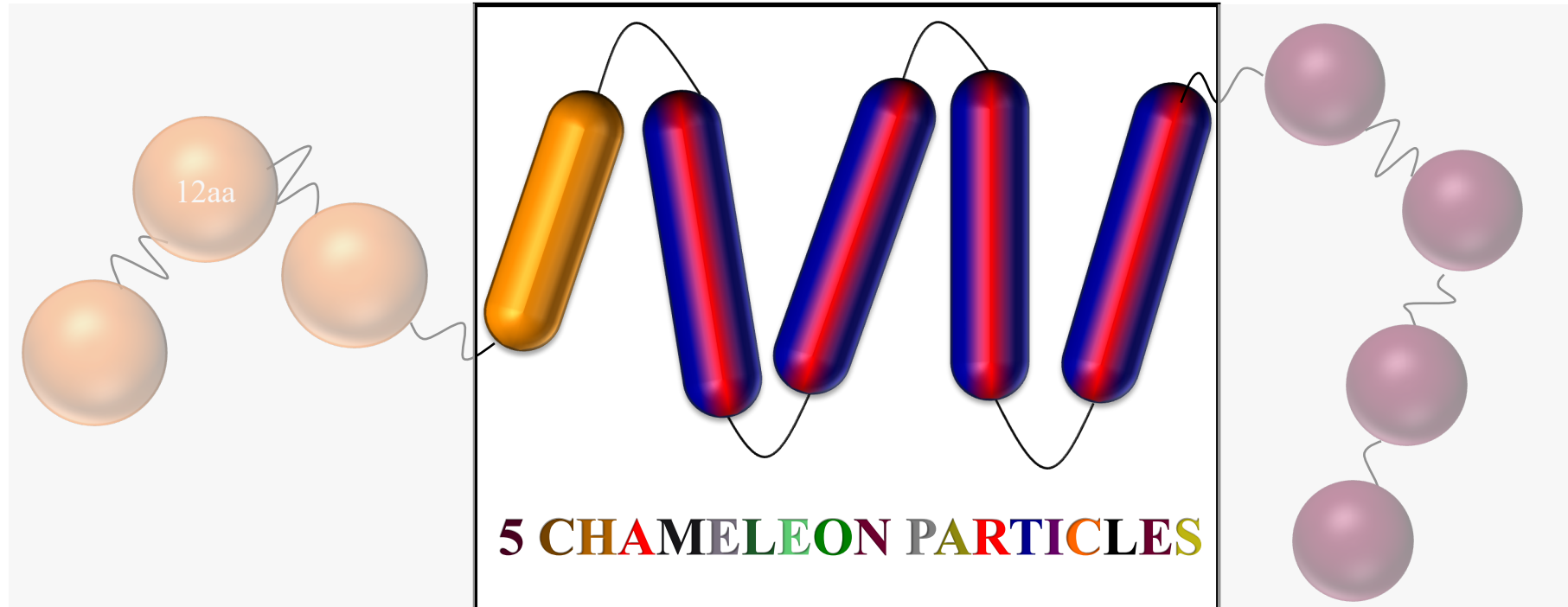
1

60-65

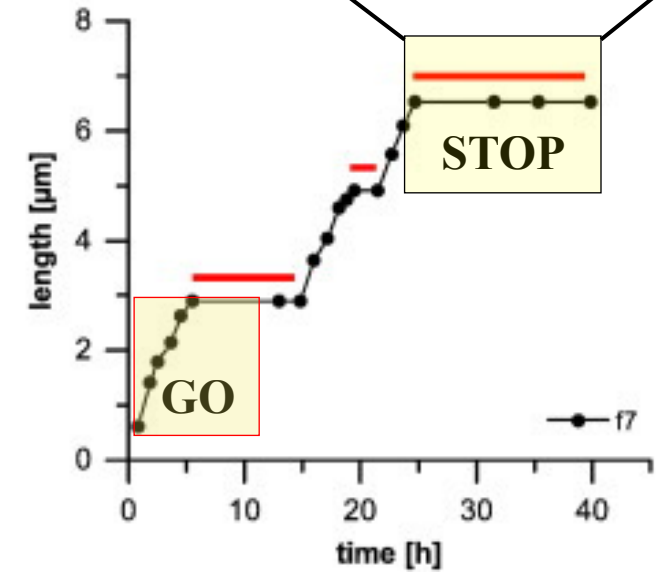
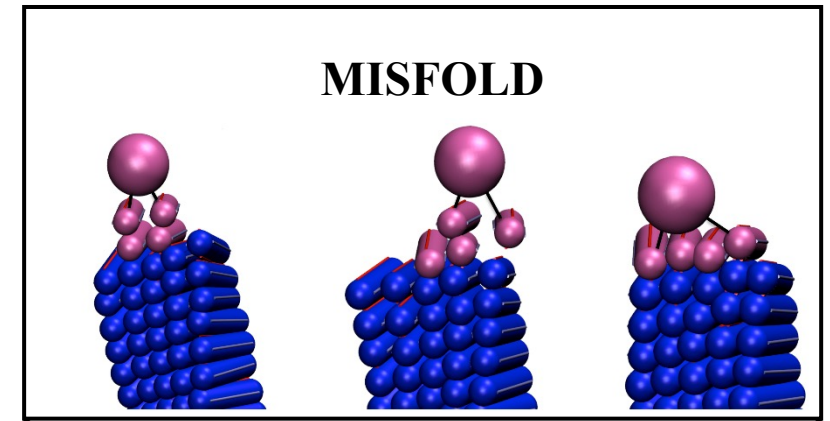
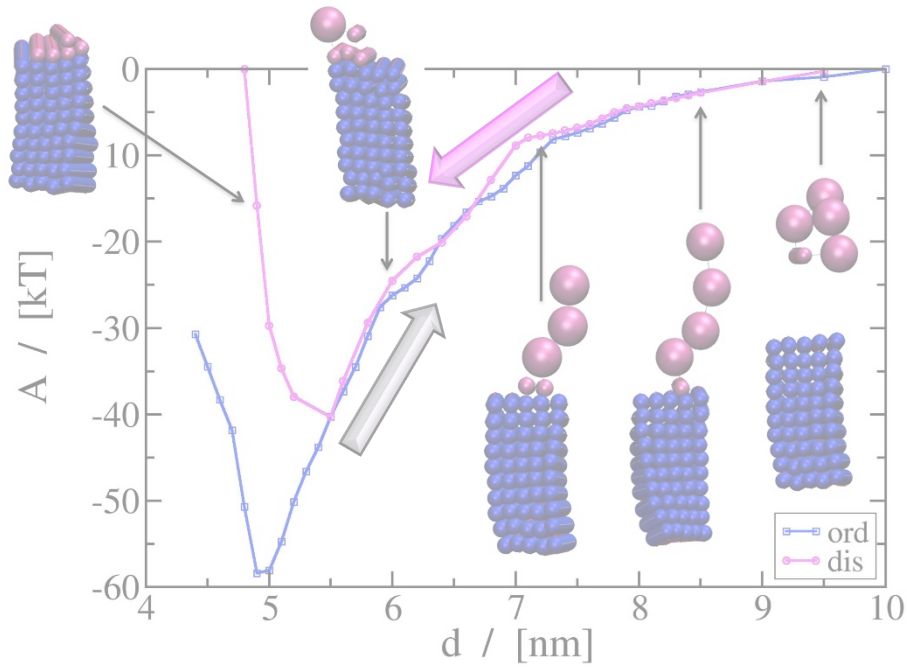
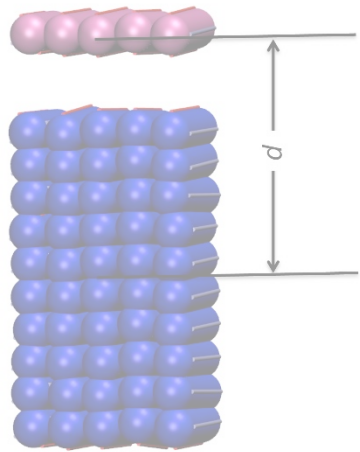
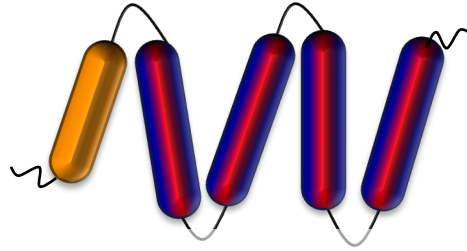
90-95

140

N-terminus	NAC region	C-terminus
<ul style="list-style-type: none"> - amphipathic (both hydrophilic and lipophilic) - α-helix, disordered 	<ul style="list-style-type: none"> - hydrophobic - building block for α-synuclein aggregates - α-helix, β-sheets, disordered 	<ul style="list-style-type: none"> - highly acidic - negatively charged - disordered



Question: Fibrillar growth



**Woerdehoff et al. JMB 427:1428-1435 (2015)

